Laser Wakefield Accelerators

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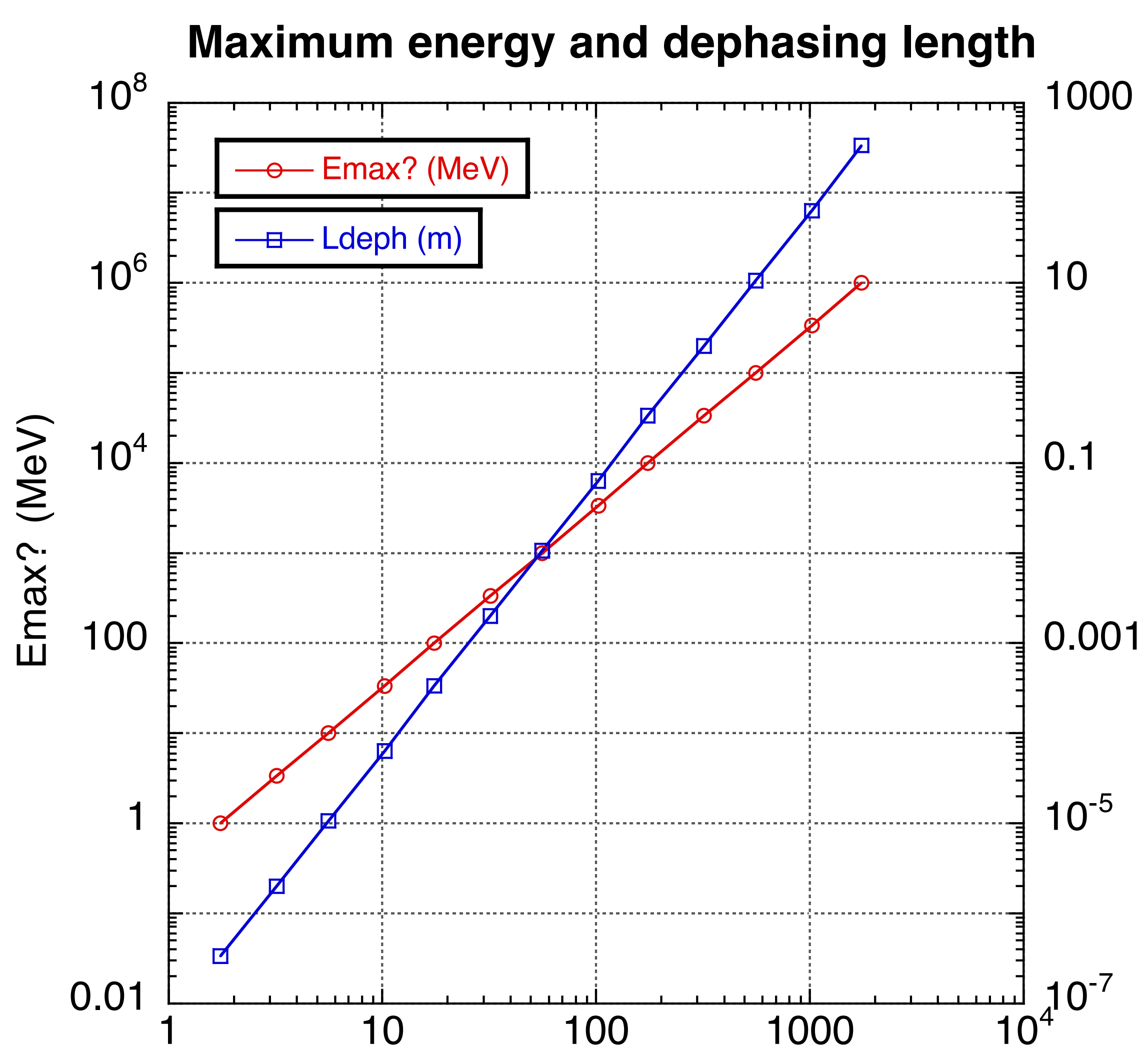
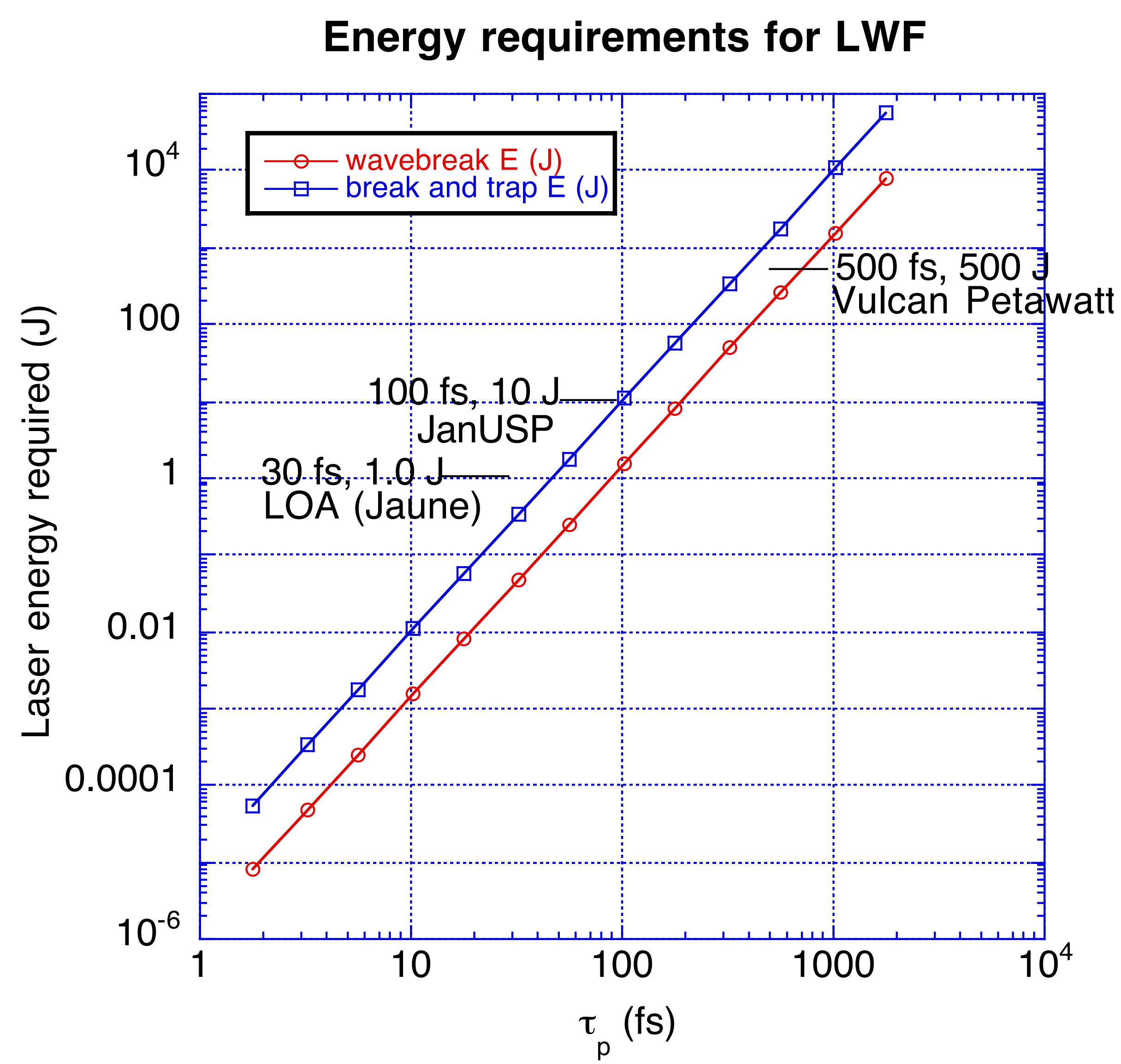
Energy gain and length

$$W_{max} \simeq 2mc^2 \left(\frac{n_{cr}}{n_e} \right) \cdot \alpha \qquad L_{deph} \simeq \lambda_0 \left(\frac{n_{cr}}{n_e} \right)^{3/2} \cdot \beta$$





Gaussian Focussing

Starting from the wave equation:

$$\nabla^{\mathbf{2}}\mathbf{E} - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$$

For a solution of the form (fast oscillations only in *z*):

$$\mathbf{E} = E(x, y, z)\exp(i(kz - \omega t))\hat{\mathbf{x}}$$

Leads to paraxial wave equation:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)E - 2ik\frac{\partial E}{\partial z} = 0$$

Gaussian Focussing

$$\frac{\partial^2 E}{\partial r^2} + \frac{1}{r}\frac{\partial E}{\partial r} - 2ik\frac{\partial E}{\partial z} = 0$$

Gaussian envelope solution

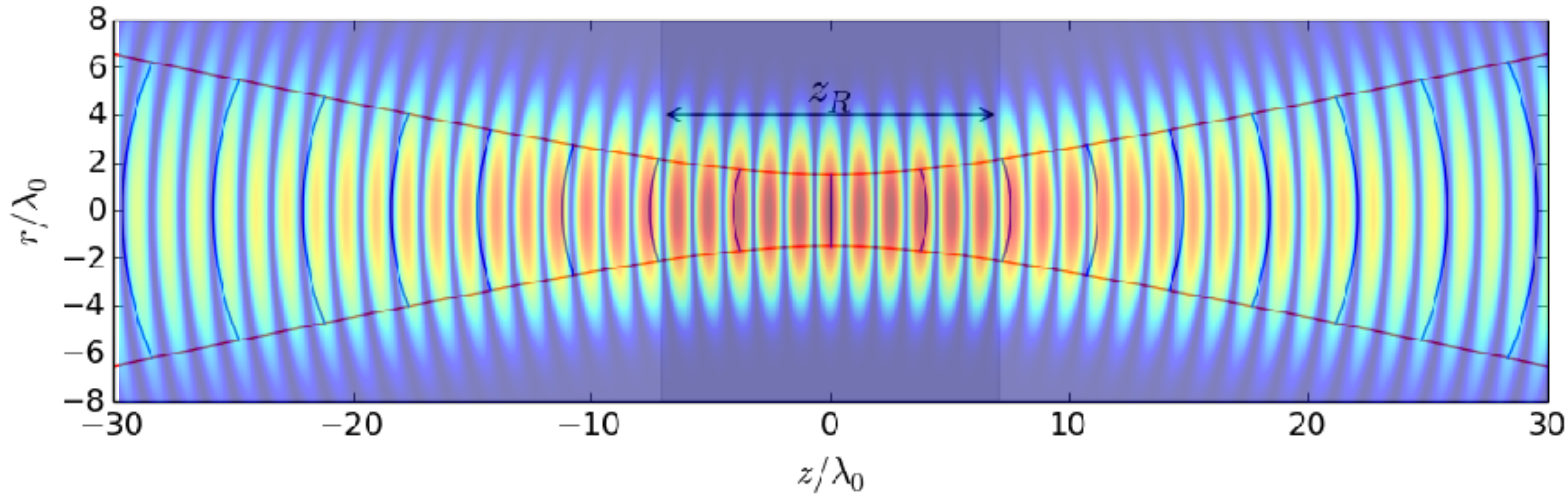
$$E(r,z)/E_0 = \frac{w_0}{w}\exp\left[\frac{-r^2}{w^2} - \frac{i\pi r^2}{\lambda R} + i\phi_0\right]$$

$$w = w_0\sqrt{1+\left(\frac{z}{z_R}\right)^2} \qquad \text{(beam waist)}$$

$$R = \frac{1}{z}\left(z^2 + {z_R}^2\right) \qquad \text{(radius of curvature)}$$

$$\tan\phi_0 = \frac{\lambda z}{\pi {w_0}^2} \qquad \text{(Gouy phase)}$$

Gaussian Focussing



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$$z_R = \frac{\pi {w_0}^2}{\lambda} \quad \text{(Rayleigh Range)}$$

Gaussian Focussing

$$I(r,z)/I_0 = \frac{w_0^2}{w^2} \exp\left[\frac{-2r^2}{w^2}\right]$$

At $z = z_R$, $I = \frac{1}{2} I_0$, so z_R is effective interaction length

For $w_0 \sim 30$ μm, $\lambda_0 \sim 1$ μm, $z_R \sim 3$ mm

But for $n_e \sim 10^{18}$ cm^{-3}, ($\lambda_p \sim 30$ μm), $L_{deph} \sim 3$ cm

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Plasma Propagation

$$\nabla^2 \mathbf{E} - \frac{\eta^2}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$$

density relativity

$$\eta_R \simeq 1 - \frac{{\omega_p}^2}{2\omega^2}\frac{n(r)}{n_0\gamma_r} \simeq 1 - \frac{{\omega_p}^2}{2\omega^2}\left(1 + \frac{\delta n}{n_0} - \frac{a^2}{2}\right)$$

where we used $\gamma = \sqrt{1 + {a_0}^2}$

For a gaussian pulse of beam width *R*

defocusing focusing

$$\frac{d^2 R}{dz^2} = \frac{1}{{z_R}^2 R^3}\left(1 - \frac{P}{P_c}\right).$$

$$P_{cr} \simeq 17\,(n_e/n_{cr})\,\mathrm{GW}$$

P. Sprangle et al, PRL, **59**, 202 (1987)

Plasma Propagation

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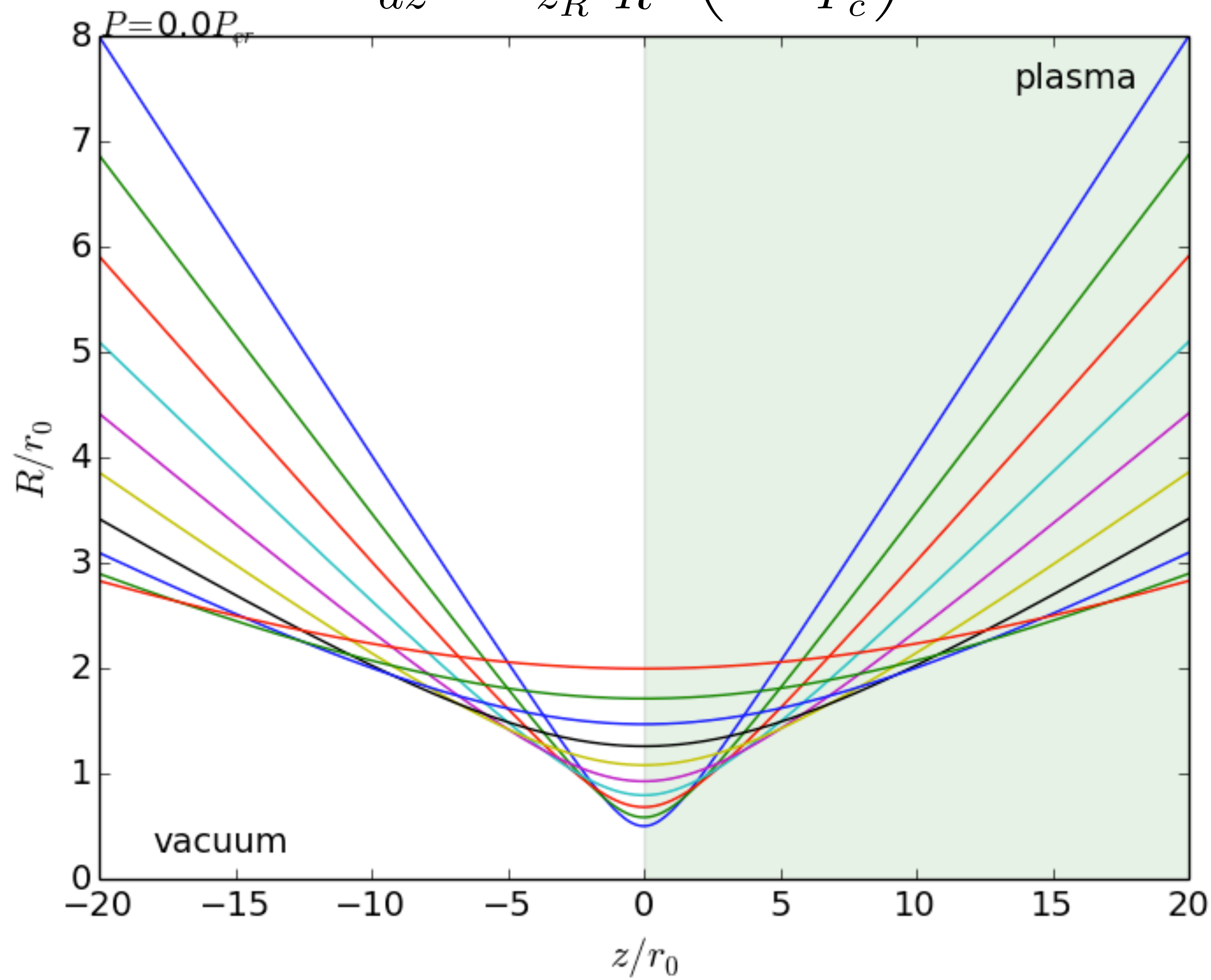

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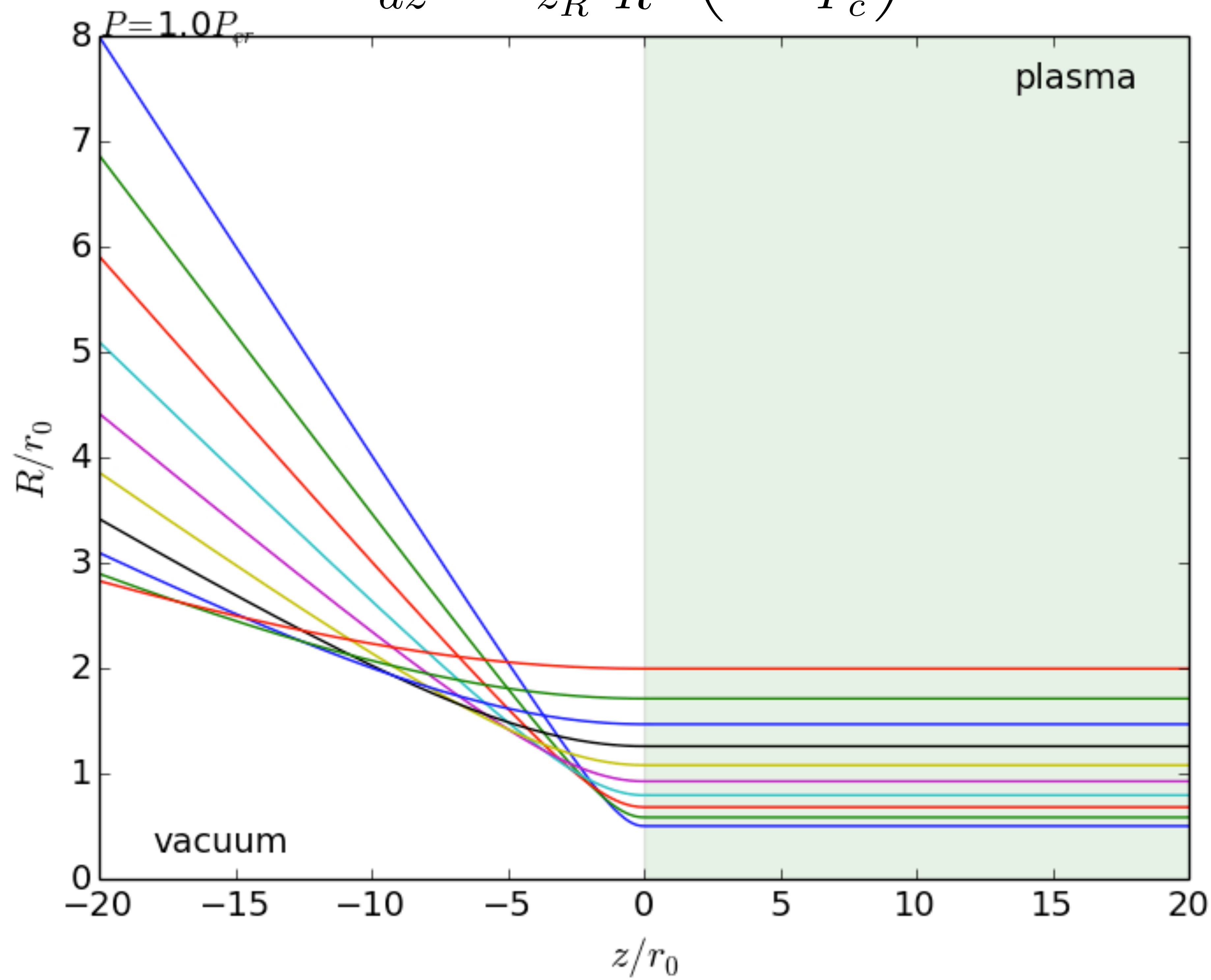
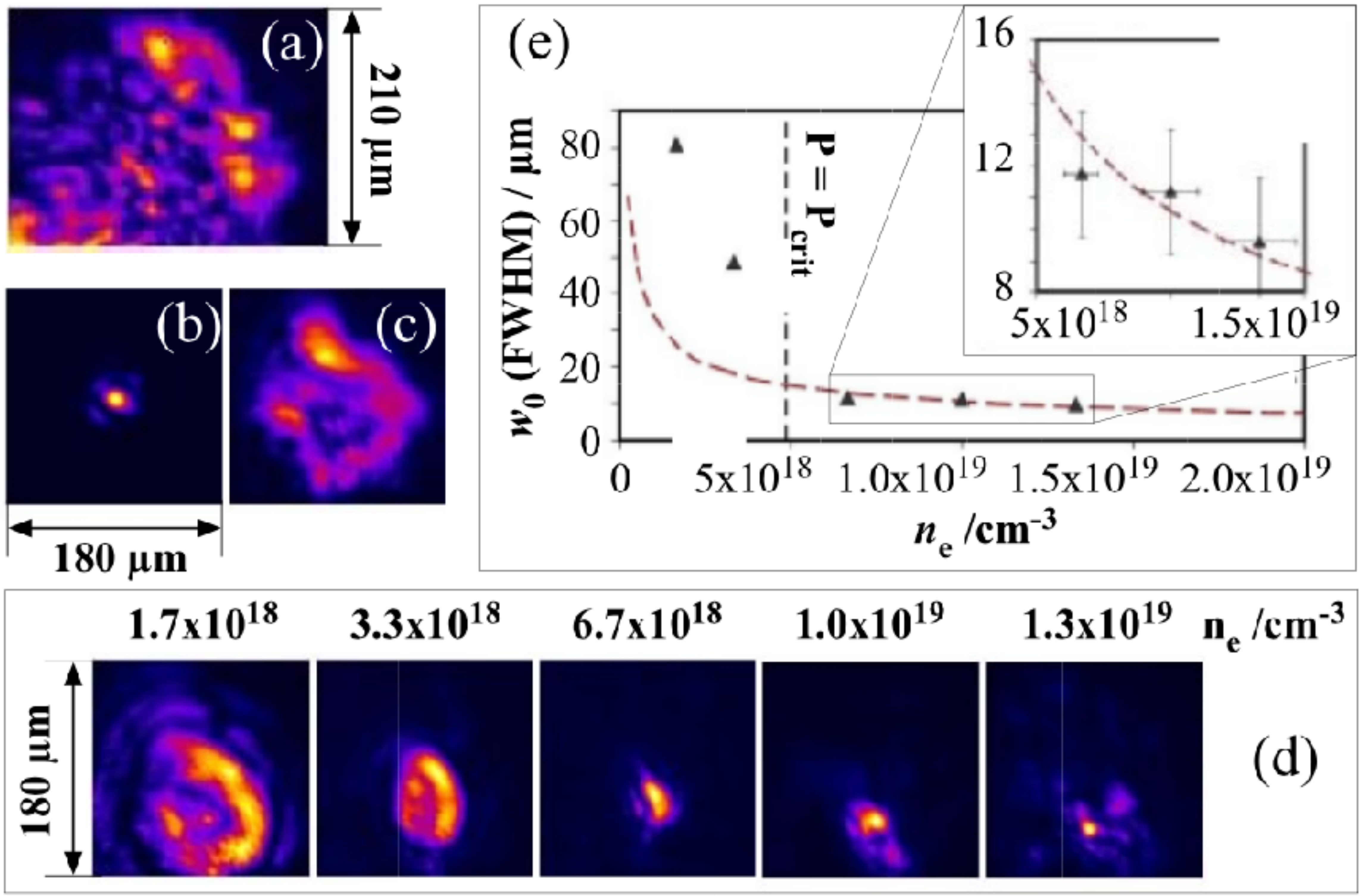


Plasma Propagation

f/3

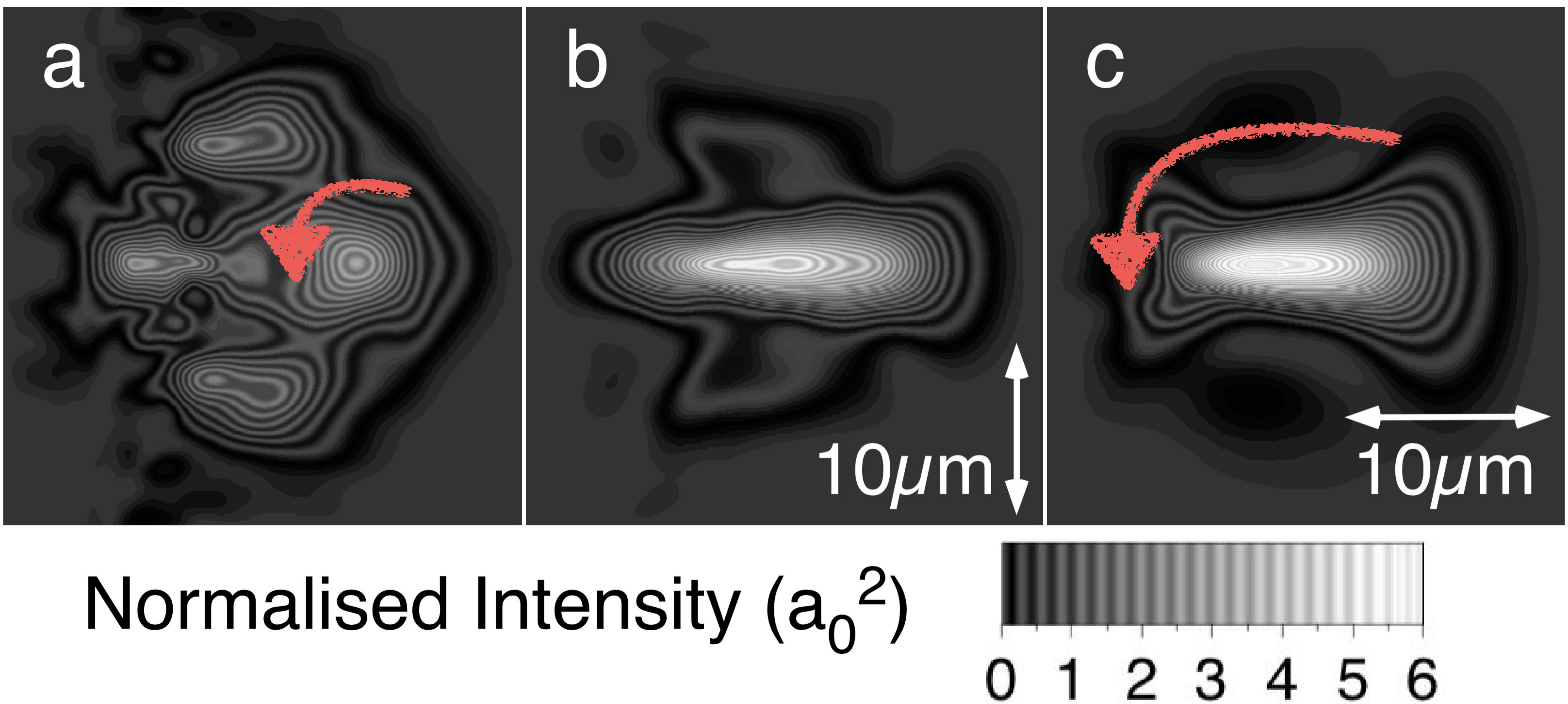
f/16



Thomas et al., PRL 98, 095004 (2007)

Plasma Propagation

Focal spots after 3 vacuum Rayleigh lengths



Thomas et al., PRL 98, 095004 (2007)

Plasma Propagation

Focussing in a guiding channel can be modelled with:

$$\frac{d^2 R}{dz^2} = \frac{1}{{z_R}^2 R^3}\left(1 - \frac{\delta n}{\delta n_c} R^4\right)$$

defocusing focusing

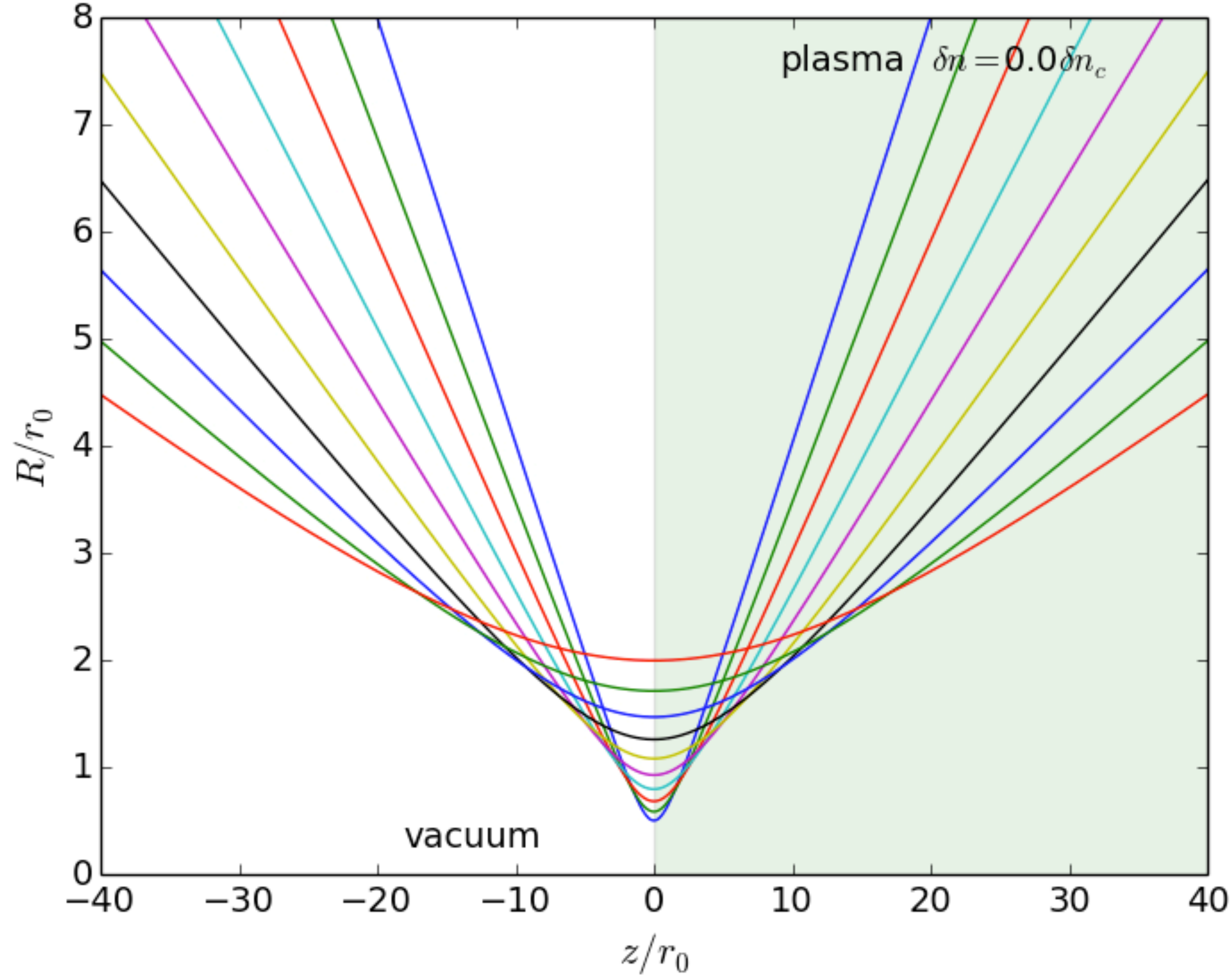
where the critical channel depth is defined by:

$$\delta n_c = \frac{1}{\pi r_e {r_0}^2}$$

here r_e is the classical radius of an electron $r_e = e^2/{m_e}^2 c^2$

Plasma Propagation

$$\frac{d^2R}{dz^2} = \frac{1}{{z_R}^2R^3}\left(1 - \frac{\delta n}{\delta n_c}R^4\right)$$



Plasma Propagation

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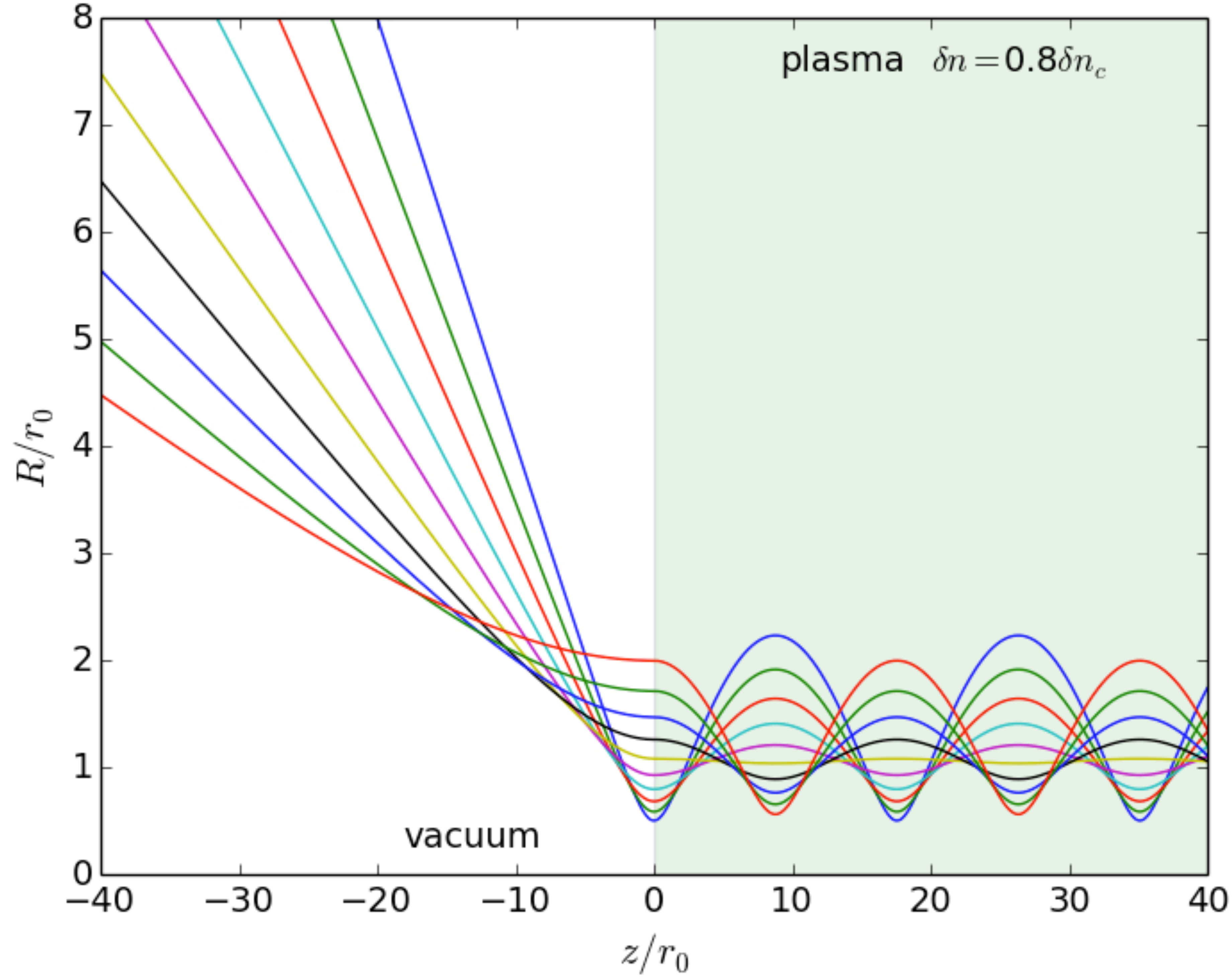
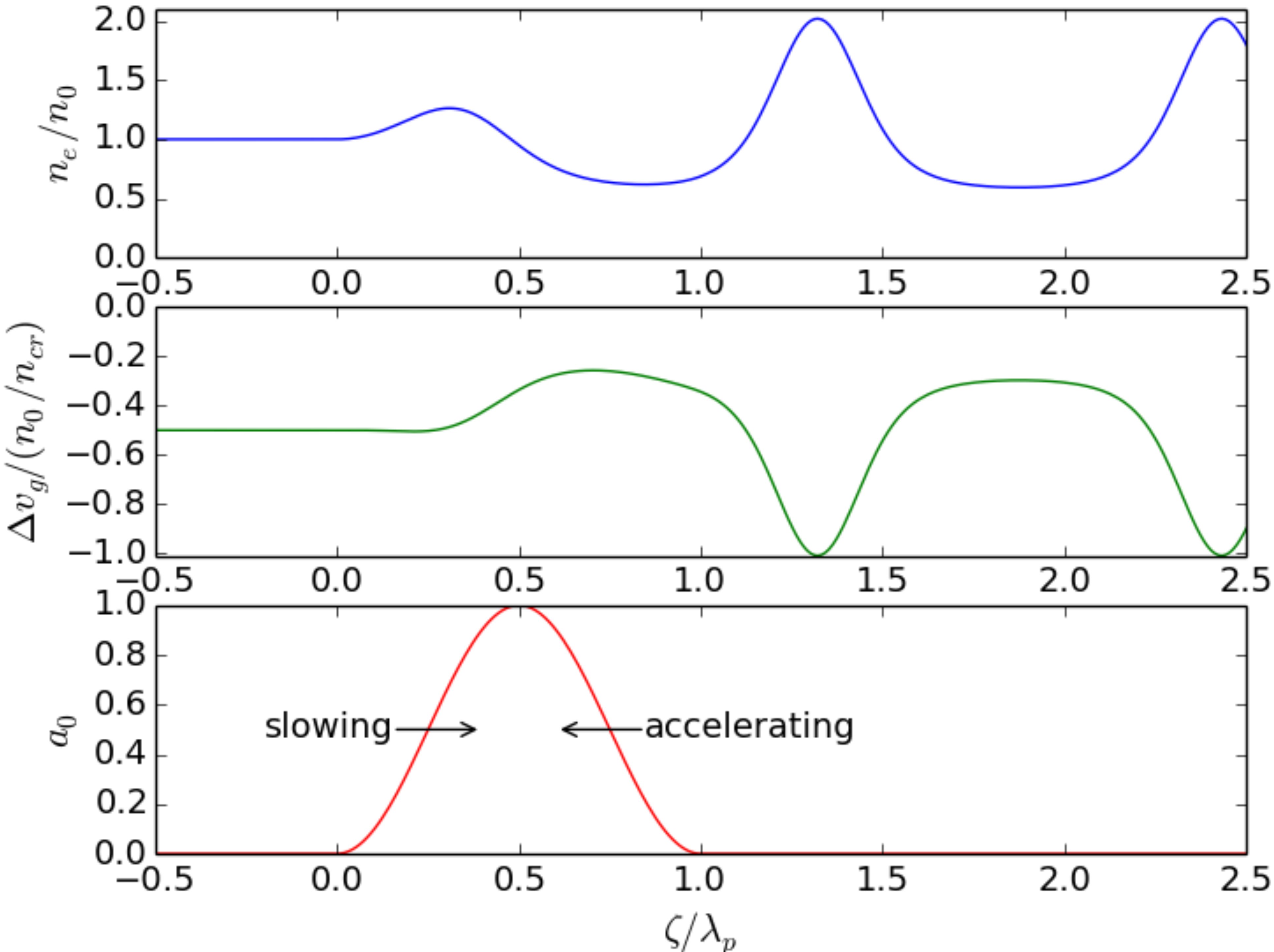


Pulse compression

$$\tau = \tau_0 - \frac{n_{e0} l}{2 n_{cr} c}$$



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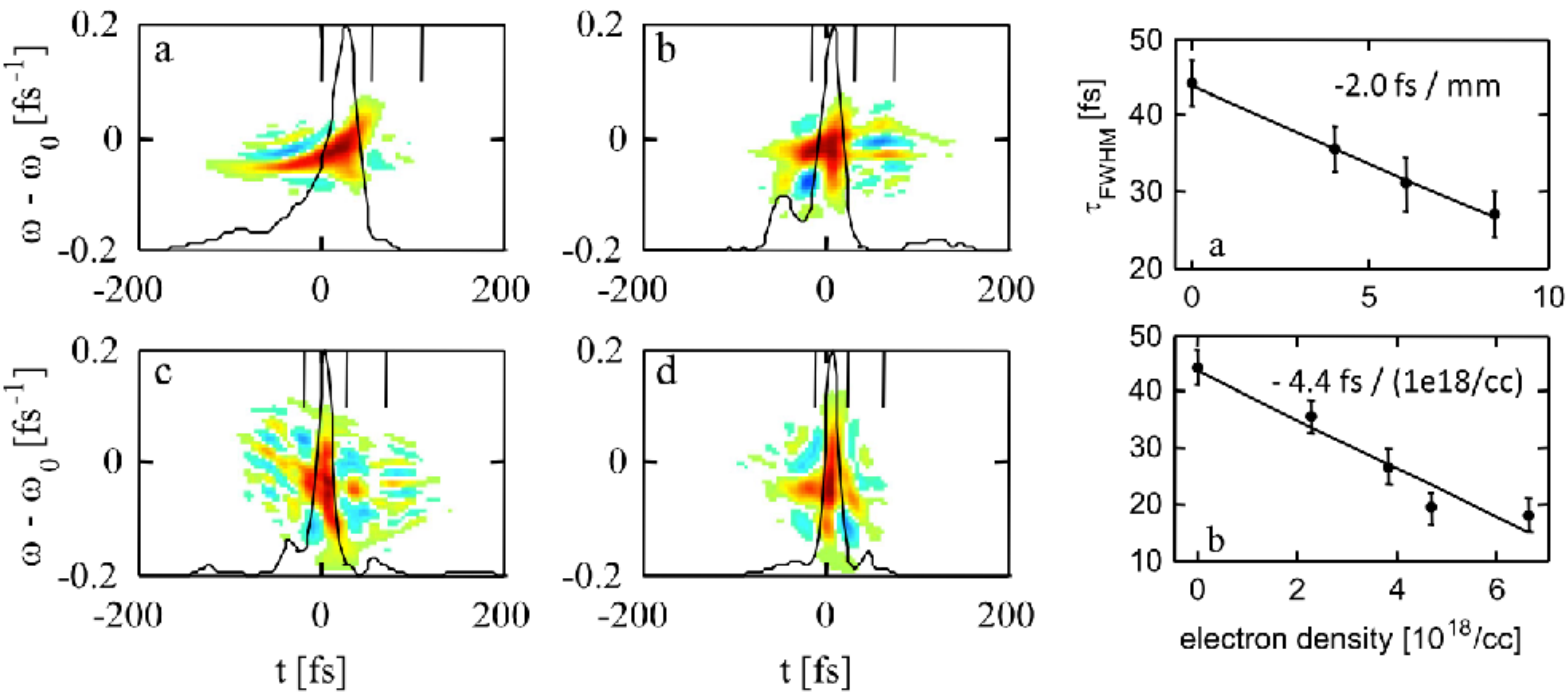
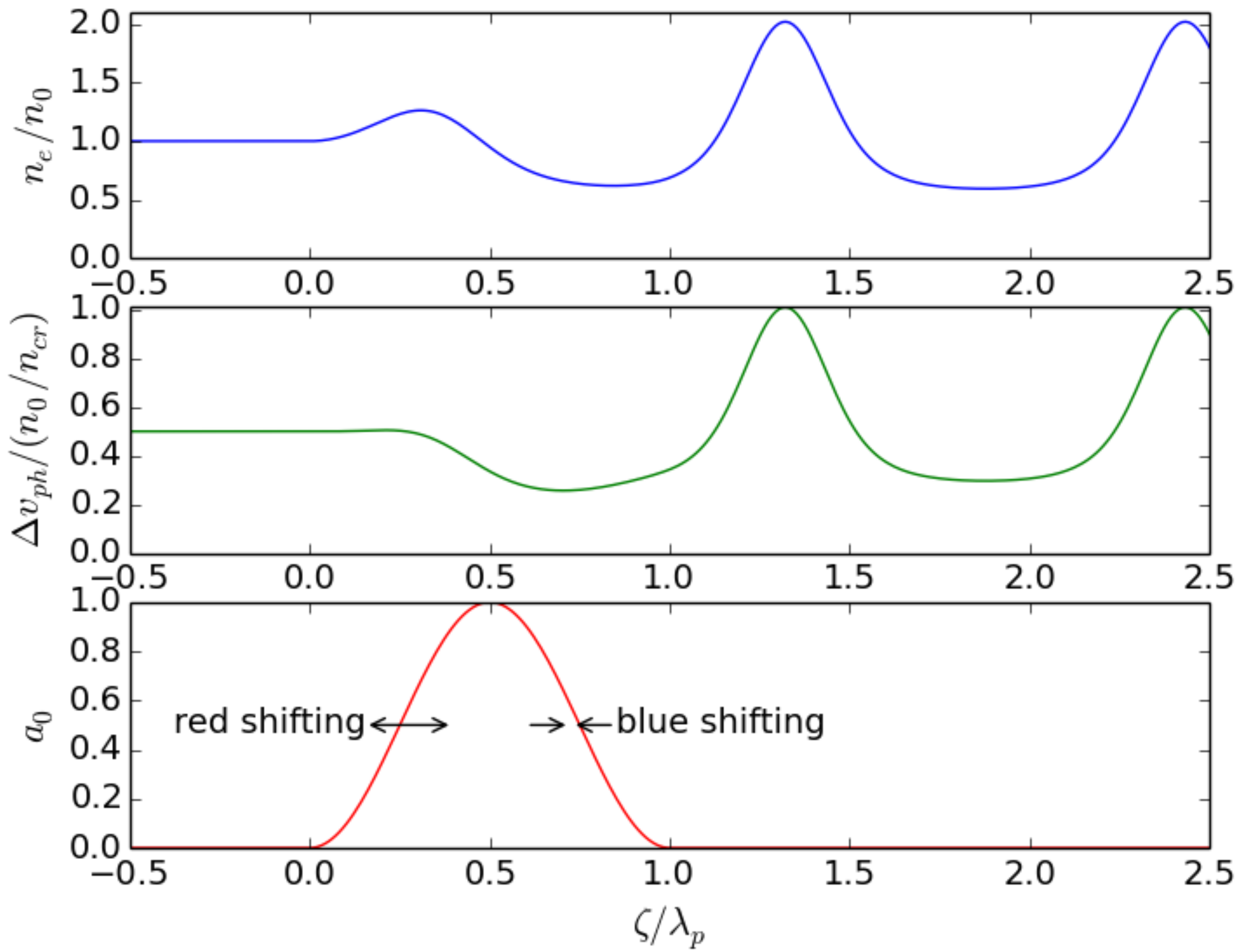


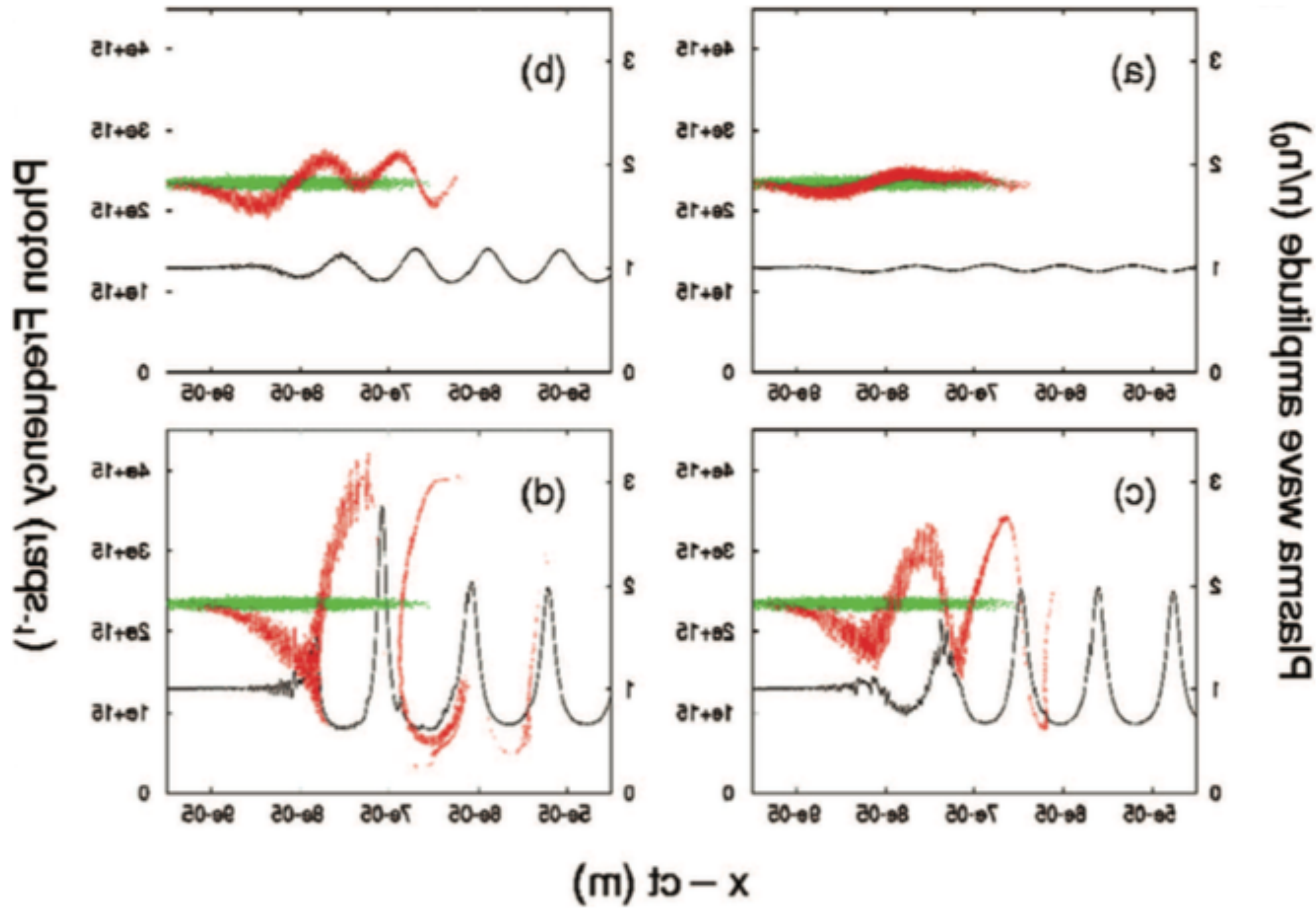
Schreiber et al., PRL 105, 235003 (2010)

Photon Acceleration

$$\delta\omega = \omega_0\left(1 - z\frac{d\beta_p}{d\zeta}\right) \simeq \omega_0\left(1 - z\frac{d}{d\zeta}\left(\frac{\delta n}{n_0}\right)\right)$$

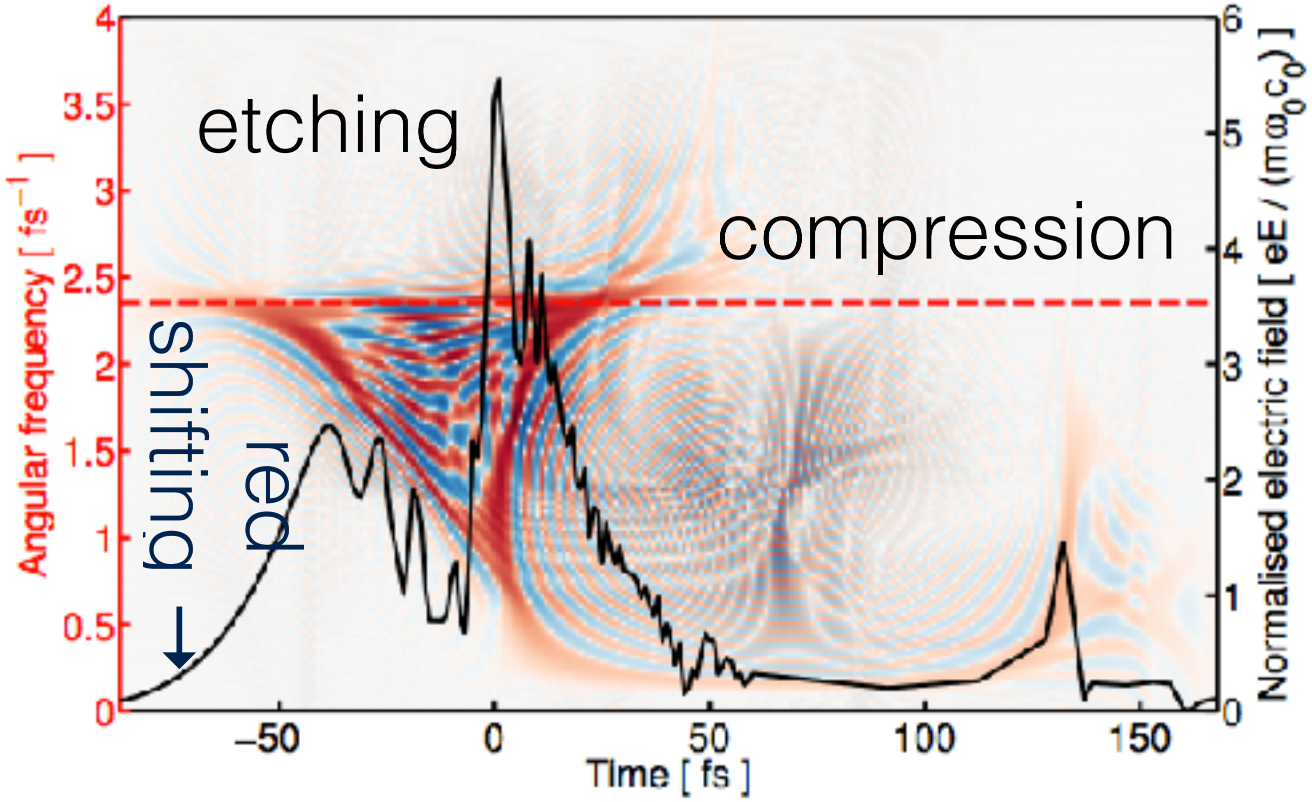


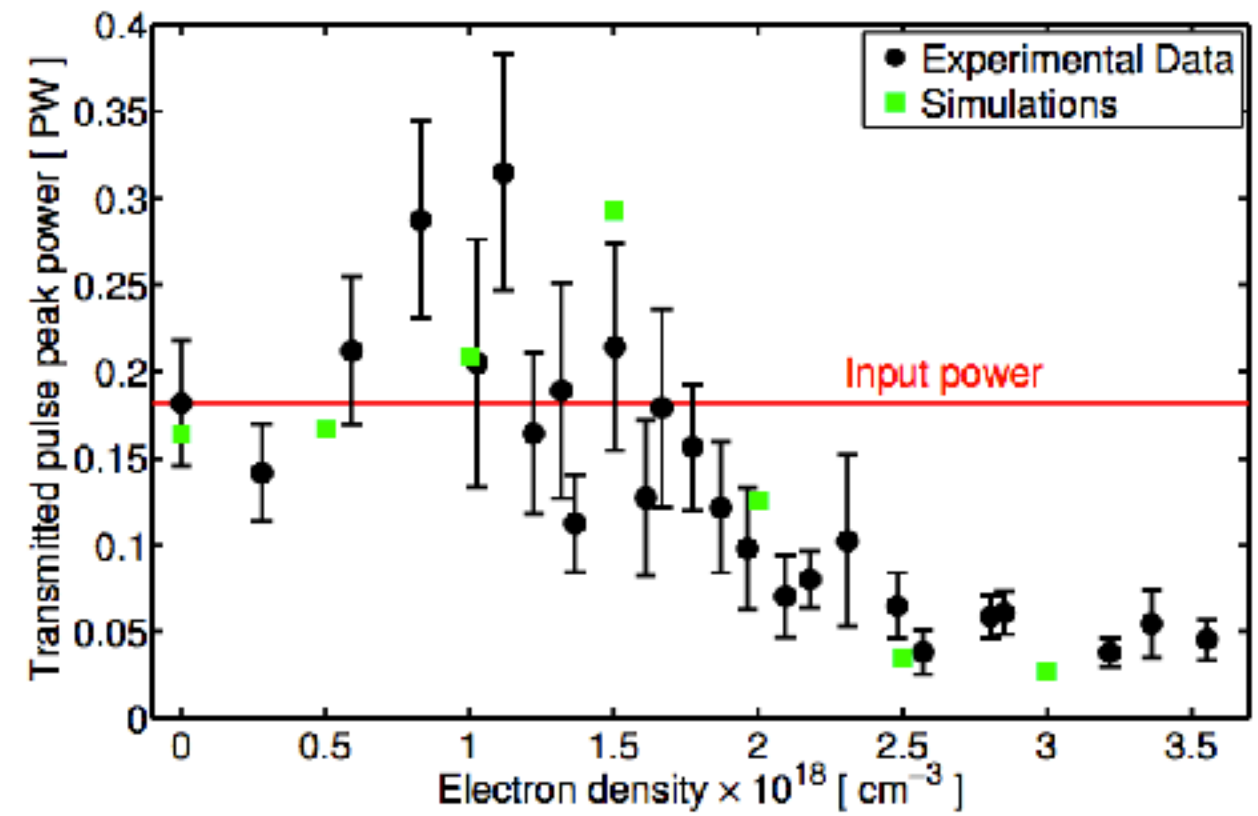
Photon Acceleration

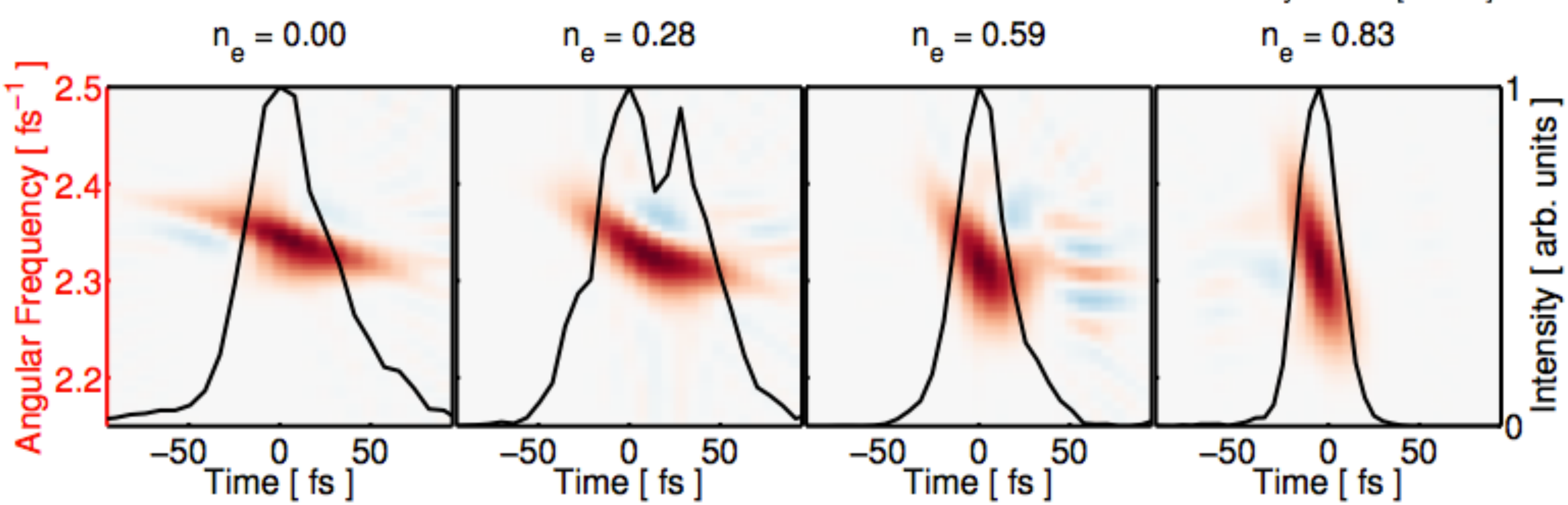


Murphy et al., POP 13, 033108 (2006)

Etching and Power







Streeter et al., In preparation (2017)

Focussing Summary

Laser pulses in vacuum only have high intensity over a Rayleigh range

Interaction can be extended for laser power $P > P_{cr}$ or by using a guiding profile $\delta n > \delta n_c$

Laser pulses lose energy to wakefield, in extreme case being etched from the front.

Compression can help maintain laser power even as laser pulse depletes.

Formula Summary

Regime	a_0	$k_p w_0$	$\delta n/n_0$	$k_p L_{deph}$	$k_p L_{depl}$	λ_W	γ_ϕ	$\Delta W/mc^2$
Linear:	< 1	2π	${a_0}^2$	$\frac{{\omega_0}^2}{{\omega_p}^2}$	$\left(\frac{{\omega_0}^2}{{\omega_p}^2}\right)\left(\frac{\omega_p \tau}{{a_0}^2}\right)$	$\frac{2\pi}{k_p}$	$\frac{\omega_0}{\omega_p}$	${a_0}^2\left(\frac{{\omega_0}^2}{{\omega_p}^2}\right)$
1D NL:	> 1	2π	a_0	$4{a_0}^2\left(\frac{{\omega_0}^2}{{\omega_p}^2}\right)$	$\frac{1}{3}\left(\frac{{\omega_0}^2}{{\omega_p}^2}\right)\omega_p \tau$	$\frac{4a_0}{k_p}$	$\sqrt{a_0}\left(\frac{\omega_0}{\omega_p}\right)$	$4{a_0}^2\left(\frac{{\omega_0}^2}{{\omega_p}^2}\right)$
3D NL:	> 2	$2\sqrt{a_0}$	$\frac{1}{2}\sqrt{a_0}$	$\frac{4}{3}\sqrt{a_0}\left(\frac{{\omega_0}^2}{{\omega_p}^2}\right)$	$\left(\frac{{\omega_0}^2}{{\omega_p}^2}\right)\omega_p \tau$	$\frac{2\pi\sqrt{a_0}}{k_p}$	$\frac{1}{\sqrt{3}}\left(\frac{\omega_0}{\omega_p}\right)$	$\frac{2}{3}{a_0}^2\left(\frac{{\omega_0}^2}{{\omega_p}^2}\right)$
Bubble:	> 20	$\sqrt{a_0}$	$\sqrt{a_0}$		$a_0\left(\frac{{\omega_0}^2}{{\omega_p}^2}\right)\omega_p \tau$			$4{a_0}^2\left(\frac{{\omega_0}^2}{{\omega_p}^2}\right)$

W. Lu et al, PR STAB 10, 061301 (2007)