# Intense lasers: high peak power
# Part 1: amplification

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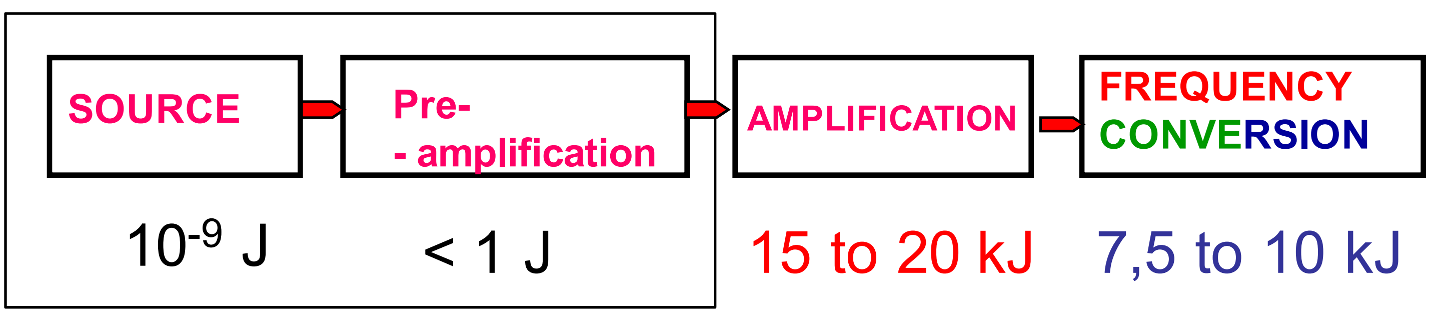
# What do you need for building a laser

:

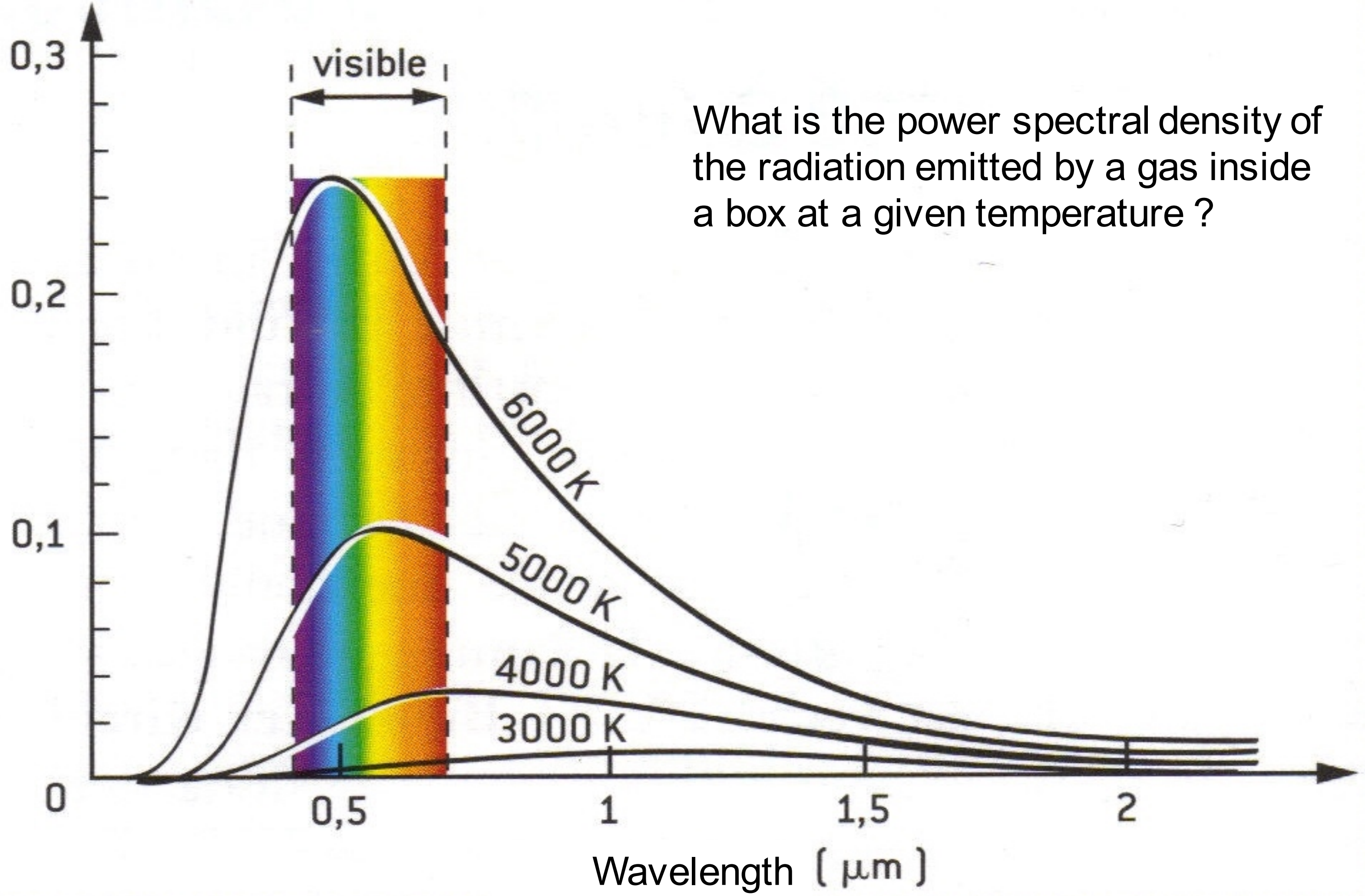
- An amplifying medium = an energy converter
- An electromagnetic radiation = an electromagnetic wave that propagates
- A resonant cavity = a set of mirrors facing each other = a « Fabry-Perot » cavity

That’s true for both an oscillator and an amplifier.

For many reasons, a Master Oscillator Power Amplifier (MOPA) is the most commonly used



# Laser-matter interaction: the Blackbody radiation




What is the power spectral density of the radiation emitted by a gas inside a box at a given temperature ?

- Semi-classical model between electromagnetic radiation and a population of atoms:
  - The spectral energy density is defined as ***ρ (ν) = U(ν) dN(ν)/ V*** the product of the average number of photons per energy mode times the photon energy times the modes density between ***ν*** and ***ν+dν***:

$$\rho(\nu)d\nu = h\nu \frac{1}{\exp(\frac{h\nu}{kT})-1} \frac{8\pi\nu^2}{c^3} d\nu$$

    - Photon energy
    - Average number of photons per mode
    - Photons density between ***ν*** and ***ν+dν***
  - That's the Planck formula from the Blackbody radiation theory

$$\rho(\nu)d\nu = \frac{1}{\exp(\frac{h\nu}{kT})-1} \frac{8\pi h\nu^3}{c^3} d\nu$$

# *Laser-matter interaction: the atomic system is made of many levels*

***Any 2, 3 or 4 level system can be seen as a 2 level system:***

- Degeneracy ***g = 2m+1*** (related to the number of sub-levels of a given kinetic momentum: orbital ***L, $m_L$***, total ***J=L+S, J, $m_J$, F=J+I, F, $m_F$)***
- Homogeneous broadening related to the lifetime of the atomic system (free atoms , electrons in the crystal field, molecules):
  - ***Lorentzian***

$$g(\omega)=\frac{\Delta}{2\pi\left[(\omega-\omega_0)^2+\Delta^2/4\right]}$$

- Inhomogeneous broadening (Doppler effect of moving atoms and molecules)
  - ***Gaussian***

$$g(\omega)=\frac{2}{\Delta}\sqrt{\frac{\ln 2}{\pi}}Exp\left(-4\ln 2\left(\frac{\omega-\omega_0}{\Delta}\right)^2\right)$$

***Δ is the Full Width at Half Maximum (FWHM) of the line shape when Δ = 1/$T_{rad}$ +1/$T_{non\ rad}$*** and

$$\int g(\omega)d\omega=1$$

# Emission, Fluorescence, Phosphorescence

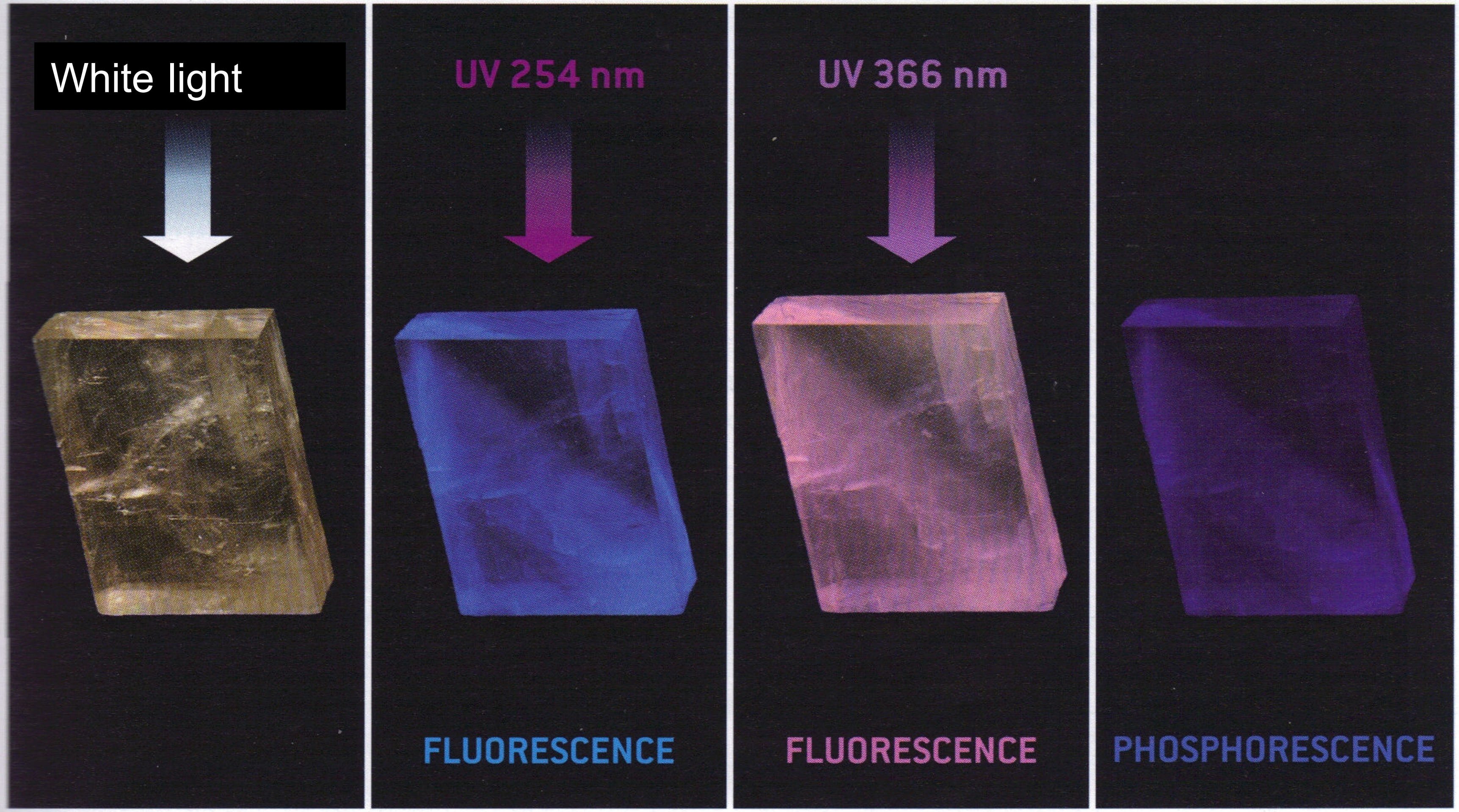
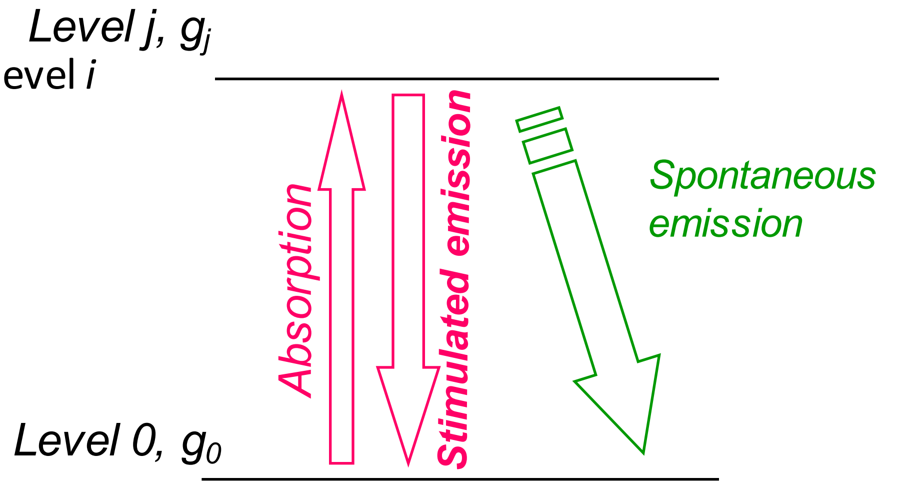



**Fig. 5** Un cristal de calcite placé sous une lampe UV émet de la fluorescence dont la couleur n'est pas la même selon la longueur d'onde d'excitation (254 nm ou 366 nm). Après suppression du rayonnement UV, l'émission de lumière persiste : il s'agit de phosphorescence.

# *Einstein's coefficients (1)*

At thermodynamic equilibrium, each process going "down" must be balanced exactly by that going "up" and the transition probability can be written:

$$P_{0\to j}=B_{0j}\rho(\nu)t$$




- $N_j$ the population (or population density) of level *i*
- $N = N_0 + N_j = cste$

*Absorption*

- $\delta N_0{}^{a} = - N_0 B_{0j}\, \rho(\nu)\, \delta t$

*Stimulated or induced emission*

- $\delta N_0{}^{sti} = + N_j B_{j0}\, \rho(\nu)\, \delta t$

*Spontaneous emission*

- $\delta N_0{}^{spont} = + N_j A_{j0}\, \delta t$

- *The balance* $\Delta N_0 = \delta N_0{}^{a} + \delta N_0{}^{sti} + \delta N_0{}^{spont} = [(N_j B_{j0} - N_0 B_{0j})\, \rho(\nu) + N_j A_{j0}]\, \delta t$
- $\Delta N_j = -\Delta N_0 = [(N_0 B_{0j} - N_j B_{j0})\, \rho(\nu) - N_j A_{j0}]\, \delta t$
- *Commonly written* $dN_j/dt$, $dN_0/dt$

# *Einstein's coefficients (2)*

Relationship between the coefficients :

- *Because* $N_j \propto g_j \exp{-E_j/kT}$ *and absorption = sum of emissions*
- $g_0 B_{0j} = g_j B_{j0}$
- $A_{j0} / B_{j0} = 8\pi h\nu^3/c^3$
- $N_j$ is the population (or $n_j$ the population density) of level *i*

$$\rho(\nu)\, d\nu = \rho(\omega)\, d\omega \text{ then } \rho(\omega) = \rho(\nu)/2\pi$$

- $B_{j0}\, \rho(\nu) / A_{j0} = n(\nu)$ *is the number of photons per mode*

  $\rho(\nu) = 8\pi\nu^2/c^3\ h\nu\, n(\nu)$ *with thermal radiation included then*

  $n(\nu) = 1/\mathrm{Exp}(h\nu/kT)-1$ *one finds* $h\nu >> kT$ *in the optical domain and stimulated/spontaneous* $\approx \mathrm{Exp}{-h\nu/kT}$ *while in the thermal domain* $h\nu << kT$ *and stimulated/spontaneous* $\approx kT/h\nu$

*When the refraction index is n, v = c/n and one defines the (laser) intensity as*

$$I_\nu = c\, \rho(\nu) / n$$

- $dN_j/dt = (N_0 B_{0j} - N_j B_{j0})\, \rho(\nu) - N_j A_{j0}$
- $dN_j/dt = (n I_\nu / c)\, A_{j0}\, (c^3/n^3) / 8\pi h\nu^3\, (N_0 g_j / g_0 - N_j) - N_j A_{j0}$
- $dN_j/dt = -A_{j0}\, (\lambda^2 / 8\pi n^2)(I_\nu / h\nu)(N_j - N_0 g_j / g_0) - N_j A_{j0}$

# *Einstein's approach (3)*

Transition (lifetime) broadening:

- General case $\rho(\nu)$ and $g(\nu)$ :
- A becomes $A' = A\ g(\nu)$ and B becomes $B' = B\ g(\nu)$
- Different cases to be considered :
- Narrow transition $g(\nu) << \rho(\nu)$ then $g(\nu) = \delta(\nu - \nu_0)$
- Broad transition $g(\nu) >> \rho(\nu)$ then $\rho(\nu) = I_\nu / c = I_0\, g(\nu - \nu_0) / c$

Relationship between the coefficients:

- Spontaneous emission isotropic and un-polarized
- Stimulated emission : transmitted wave has the same frequency, is in the same direction, and has the same polarization as the incident wave.
- There is a relation between gain and intensity

# *Amplification (1)*

One writes the intensity balance $\Delta I = I_{transmitted} + I_{spontaneous} - I_{incident}$ as a function of the Einstein's coefficients in a two-level atomic system (1, 2) for a given medium thickness $\Delta z$

$\Delta I = h\nu B_{21} I\nu/c\, g(\nu)\, N_2\, \Delta z$

$- h\nu B_{12} I\nu/c\, g(\nu)\, N_1\, \Delta z$

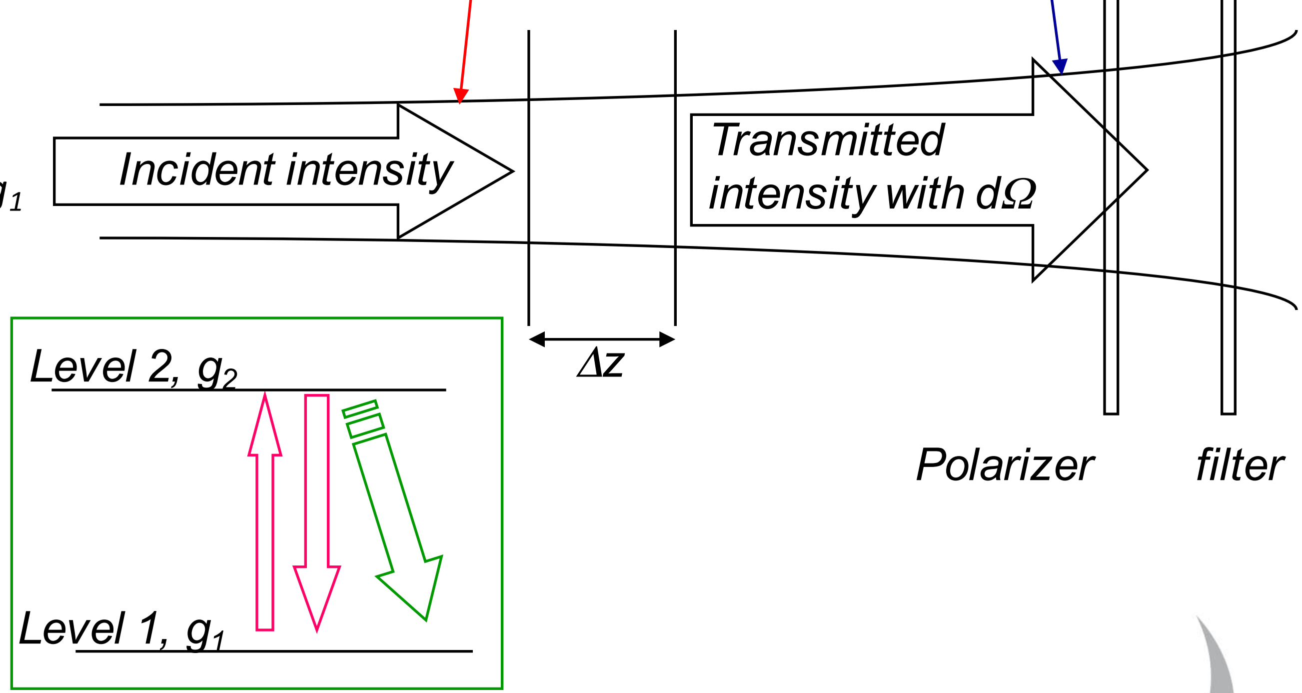
$+ h\nu A_{21}\ \Delta\nu\ g(\nu)\, N_2\, \Delta z\, d\Omega/4\pi$ *one direction* *acceptance cone*

$\Delta I / \Delta z = h\nu B_{21} (N_2 - N_1\, g_2/g_1)\, g(\nu)\, I\nu + h\nu A_{21}\ \Delta\nu\ g(\nu)\, \Delta z\, d\Omega/4\pi$

*There is gain if :*

- $N_2 > N_1\, g_2 / g_1$

*With a « noise » contribution even without incident light*



# Amplification (2)

*The gain factor reads:*

$dI\nu/dz = A_{21}(\lambda^2/\ 8\pi n^2\ )g(\nu)(N_2 - N_1\ g_2\ /\ g_1)I\nu = \gamma_0(\nu)\ I\nu$

*This* $\gamma_0(\nu)$ *or* $g_0(\nu)$ *is the small signal gain when* $I_{incident}$ *is small compared to a so-called « saturation » value* $I_{saturation}$ .

*The first part of* $dI\nu/dz$ *is the transition cross section. There is a difference between stimulated emission cross section and absorption cross section.*

$\gamma_0(\nu) = A_{21}(\lambda^2/\ 8\pi n^2\ )g(\nu)(N_2 - N_1\ g_2\ /\ g_1)$

$\sigma_{se} = A_{21}(\lambda^2/\ 8\pi n^2\ )g(\nu)$

$\sigma_{ab} = A_{21}(\lambda^2/\ 8\pi n^2\ )g(\nu)\ g_2\ /\ g_1$

$\Delta N = N_2 - N_1\ g_2\ /\ g_{1,}$ *so far :* $\boldsymbol{\gamma_0(\nu) = \sigma_{se}\ \Delta N}$

*When* $dI\nu/dz$ *can be integrated over z then:* $I\nu(z) = I\nu(0)\ Exp[\gamma_0(\nu)\ z]$

$G_0(\nu) = Exp[\gamma_0(\nu)\ z] = I\nu(z)/\ I\nu(0)$ *is the gain.*

*Another very important factor is the saturation fluence :* $\boldsymbol{F_{sat} = h\nu/\sigma}$

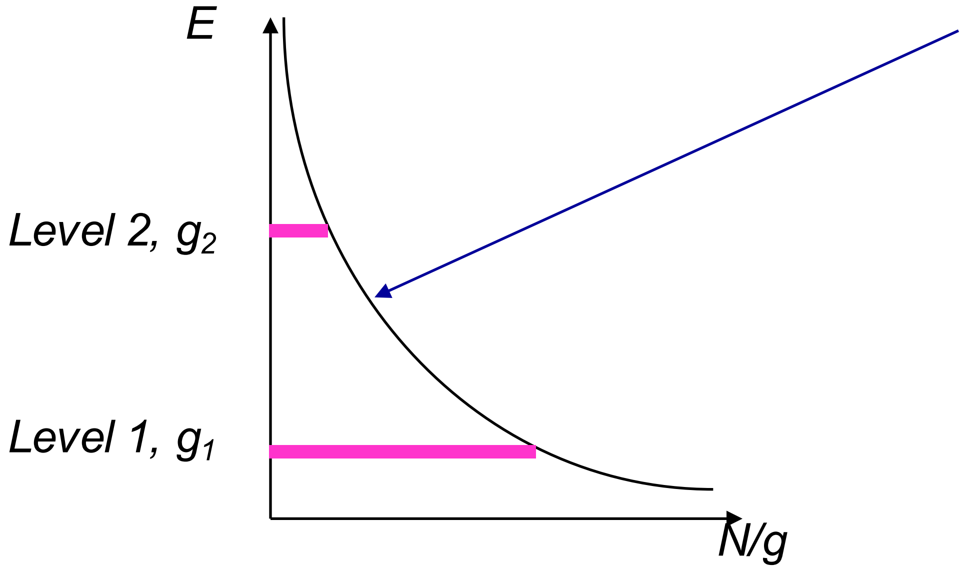
# Population inversion (1)

*As soon as : $N_2 > N_1\, g_2 / g_1$ or $\gamma_0(\nu) > 0$, there is population inversion or populations are said to be "inverted"*

*When there are relations between Einstein's coefficients or rate equations*

***Population inversion ⇔ amplification***

*At thermodynamic equilibrium, level populations are given by the Maxwell-Boltzmann relationship: $N_j \propto g_j\, exp{-}E_j/kT$*




*If E2 > E1, then $N_2/g_2 < N_1/g_1$*

*So far:*

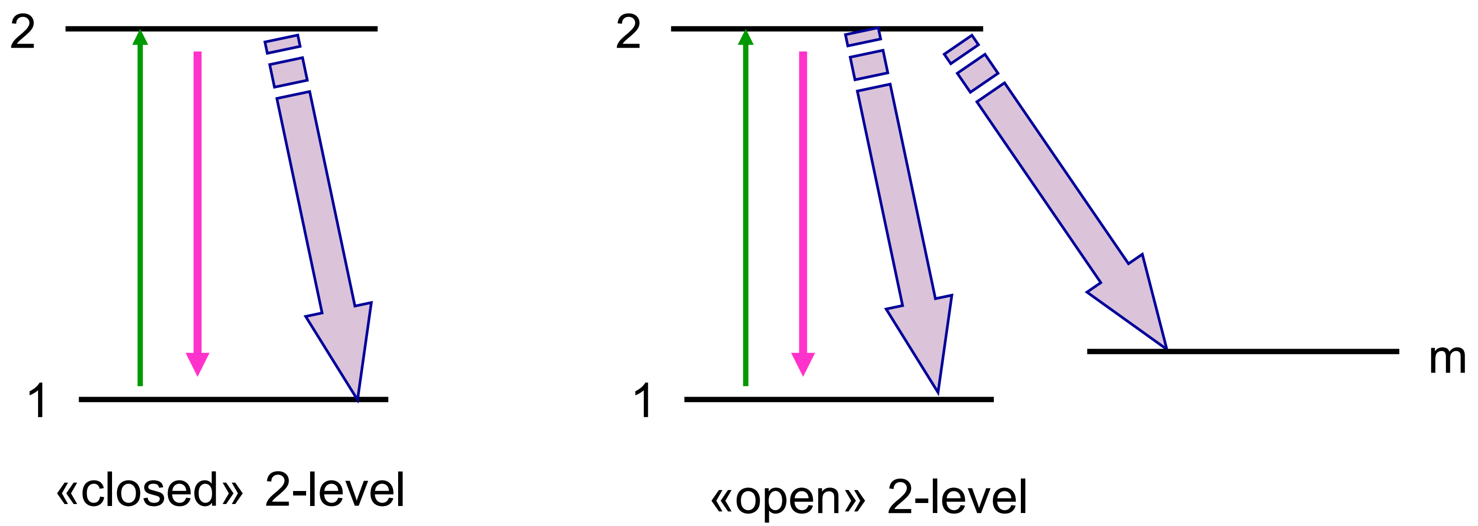
- *$N_2/g_2 > N_1/g_1$ is an abnormal state of affairs.*

*This state has to be sustained to compensate for emission losses.*

- *The extracted energy $E = \Delta N\, h\nu$ tells us that anytime 1 photon is emitted ⇔ the atom "goes" from $E_2$ to $E_1$*

# Population inversion (2) : 2-level system

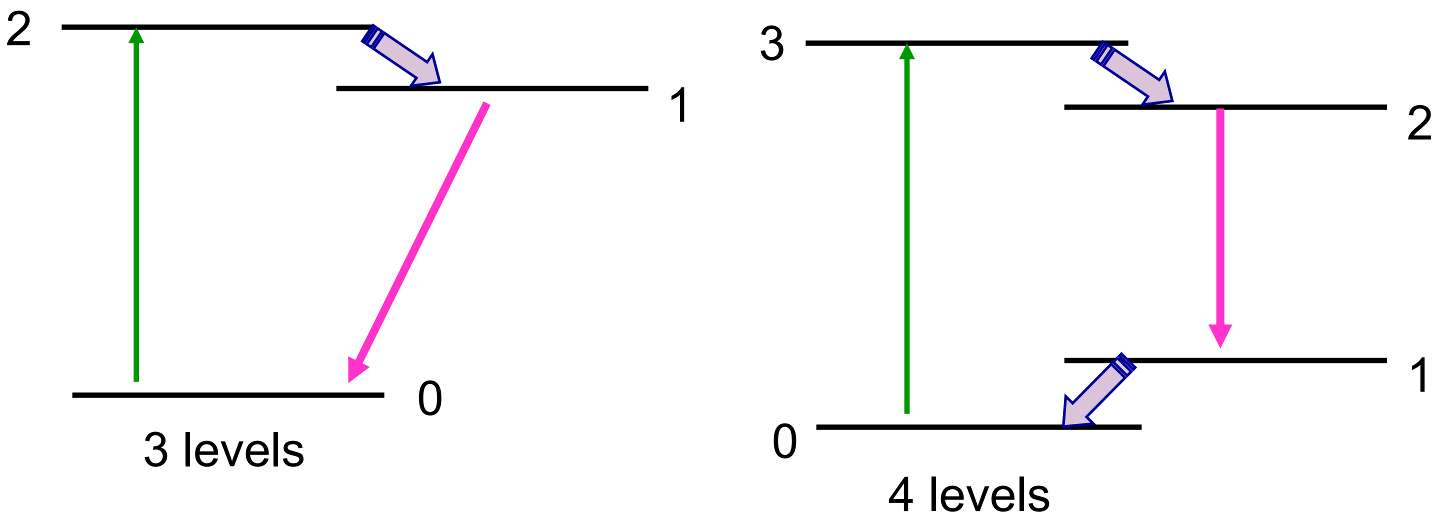
- In a 2-level system, population inversion is impossible




- The probability to empty level 2 is always greater than that to empty level 1. In the « open » case, le upper level will be progressively drained to the meta-stable level and the lower level will be "depleted": this process is called « optical pumping ».

# *Population inversion (2) : 3-level and 4-level systems*

- According to selection rules between levels (parity, ΔL, ΔJ, ΔF= 0, ±1), absorption, spontaneous or stimulated emission are or are not possible between any set of 2 levels.




- Non radiative transitions are possible: collisions (gas), crystal vibrations. These transitions can allow fast population transfers between neighbor levels.

# *The laser was born in 1960, May 16th .*

- Maiman has used a flash lamp (GE FT-506 model) inside a simple aluminum tube.
- The rod has a 0.95 cm diameter (3/8 inch) and a 1.9 cm length (3/4 inch) with end faces coated with silver.
- On one face, the central part of the silver coating is removed in order to let the radiation escape from the rod.

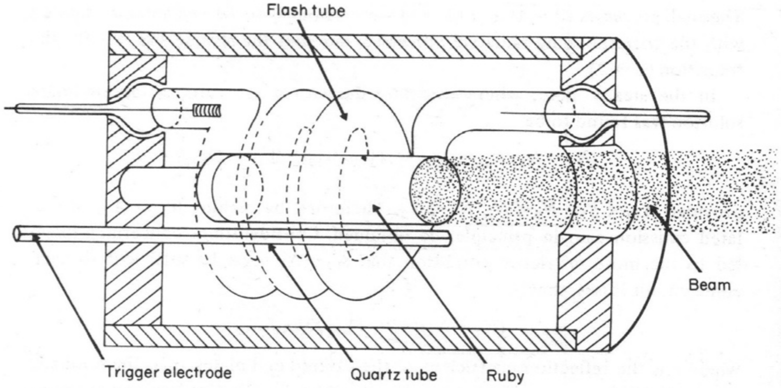
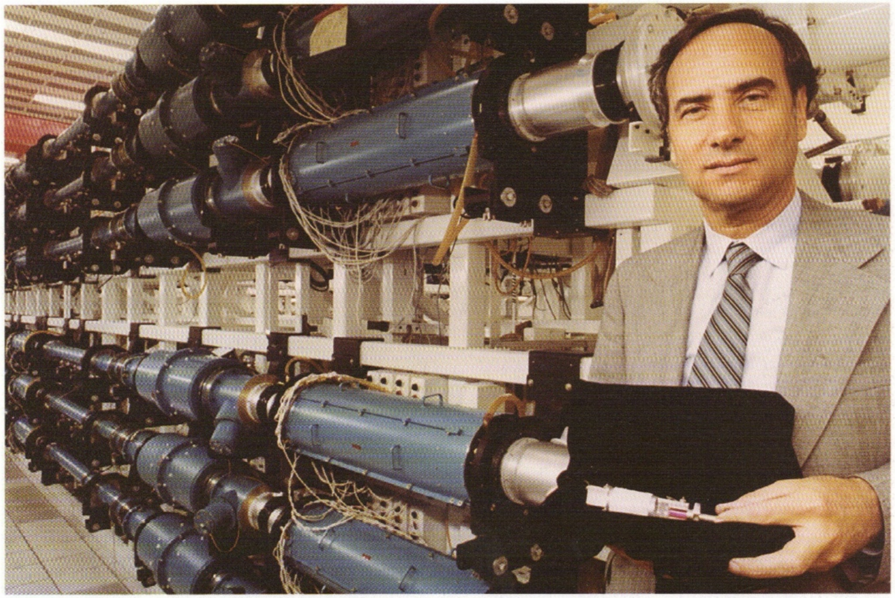



**Figure 6.4** Apparatus used by Maiman for the first ruby laser[8].



A 1980 TRW news release photo at Lawrence Livermore Labs shows the evolution of their 'Nova' from the first laser.

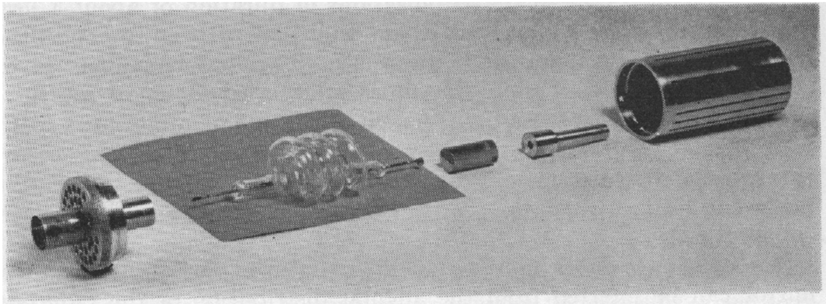


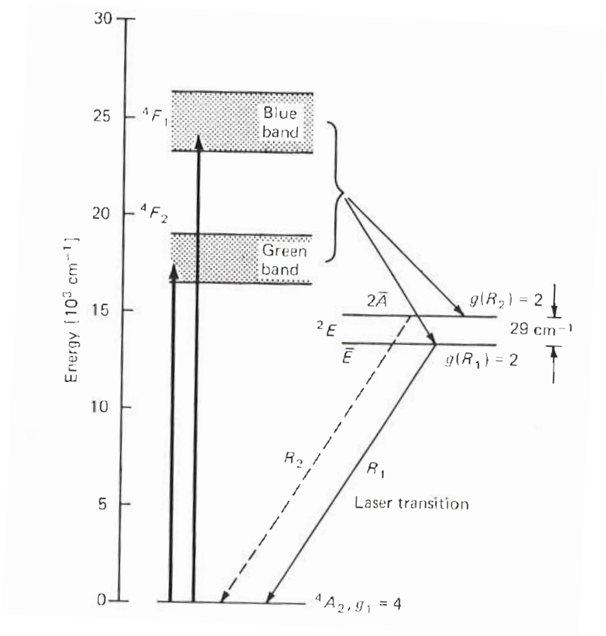
FIG. 4. Ruby laser (photograph of essential parts).

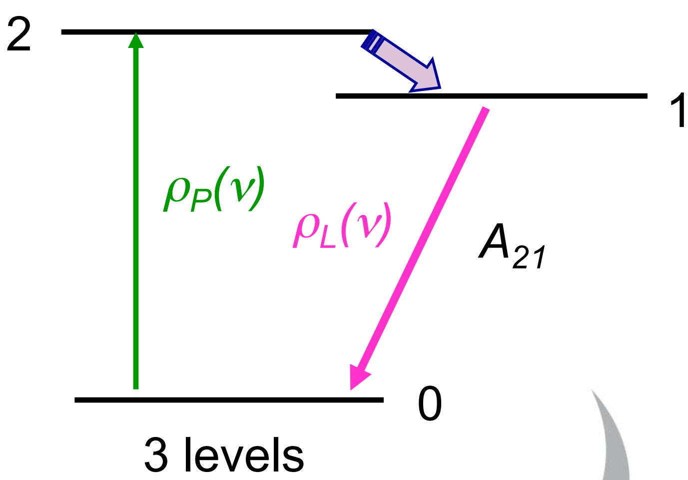
# *3-level system*

- $dN_2/dt = (N_0 B_{02} - N_2 B_{20})\, \rho_P(\nu) - N_2 A_{20} - N_2 A_{21}$
- $dN_1/dt = (N_0 B_{01} - N_1 B_{10})\, \rho_L(\nu) - N_1 A_{10} + N_2 A_{21}$
- $dN_0/dt = (-N_0 B_{01} + N_1 B_{10})\, \rho_L(\nu) + N_1 A_{10} + (-N_0 B_{02} + N_2 B_{20})\, \rho_P(\nu) + N_2 A_{20}$
- $d(N_0+N_1+N_2)/dt=0$

- *To achieve* $N_1 > N_0$ ( $N_1 > 50\%\ N_t$):
  - $dN_2/dt=0$
  - $A_{21}$ *greater than* $A_{20}$ *and* $B_{20}\, \rho_P(\nu)$

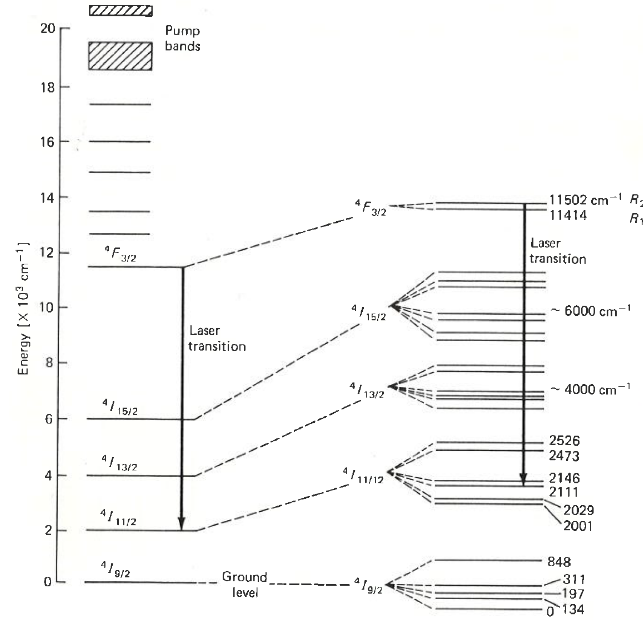
- *High* $\rho_P(\nu)$*, means high pumping intensity*
- ***3 levels:*** $Al_2O_3$ ***, Ruby laser***

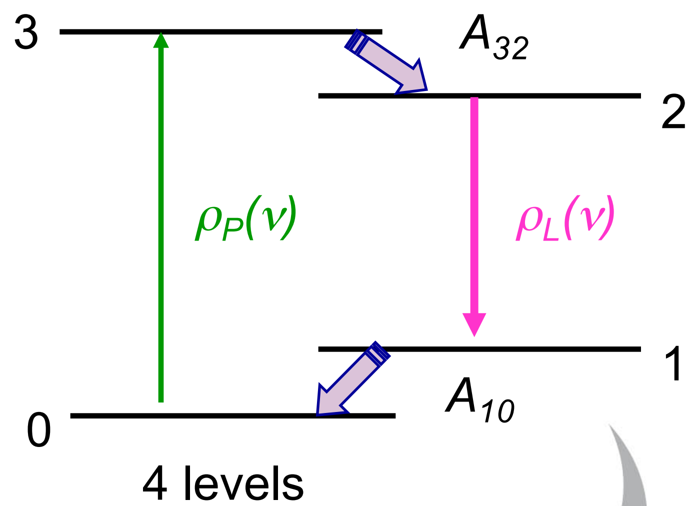


# 4-level system

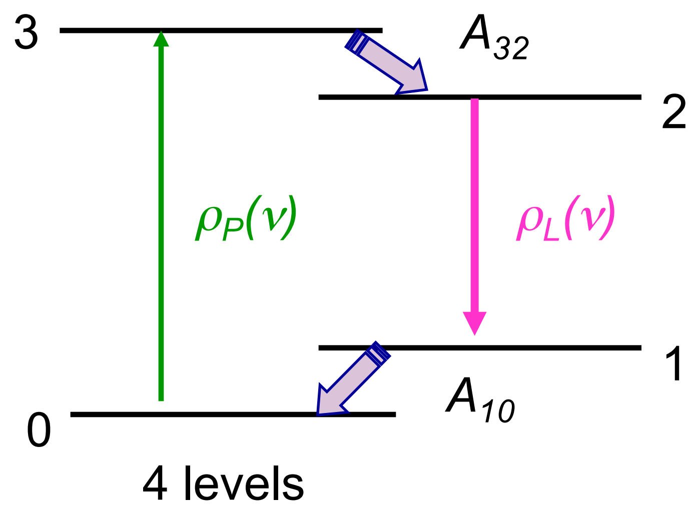
- $dN_3/dt = (N_0 B_{03} - N_3 B_{30})\, \rho_P(\nu) - N_3 A_{30} - N_3 A_{32}$
- $dN_2/dt = (N_1 B_{12} - N_2 B_{21})\, \rho_L(\nu) - N_2 A_{21} + N_3 A_{32}$
- $dN_1/dt = (-N_1 B_{12} + N_2 B_{21})\, \rho_L(\nu) + N_2 A_{21} - N_1 A_{10}$
- $dN_0/dt = (-N_0 B_{03} + N_3 B_{30})\, \rho_P(\nu) + N_3 A_{30} + N_1 A_{10}$
- $d(N_0+N_1+N_2+N_3)/dt=0$
- at $t = 0$, $N_1=N_2=0$
- If $A_{32}$ is much greater than $A_{30}$ and $B_{30}\, \rho_P(\nu)$, as soon as $N_3 \neq 0$, then $dN_2/dt = N_3 A_{32}$, so $N_2 \neq 0$
- There is always gain when $N_2>N_1$
- In order to sustain the cycle, $A_{10}$ must be large enough, otherwise a « bottleneck» effect can occur.
- **4 levels : $Nd^{3+}$ : $Y_3Al_5O_{12}$ , YAG laser (garnet)**

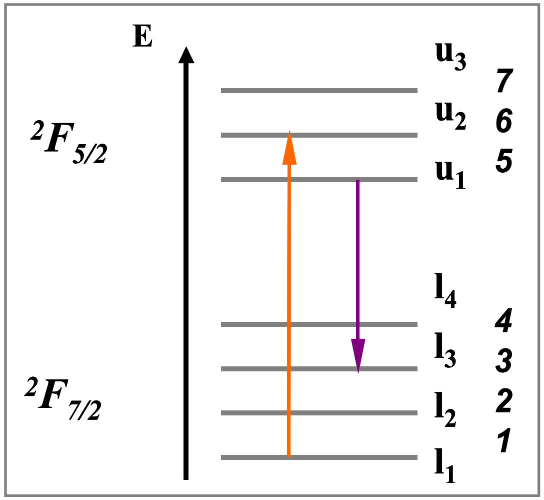


# Quasi 3-level system

- $dN_3/dt = (N_0 B_{03} - N_3 B_{30}) \rho_P(\nu) - N_3 A_{30} - N_3 A_{31} - N_3 A_{32}$
- $dN_2/dt = (N_1 B_{12} - N_2 B_{21}) \rho_L(\nu) - N_2 A_{21} - N_2 A_{20} + N_3 A_{32}$
- $dN_1/dt = (-N_1 B_{12} + N_2 B_{21}) \rho_L(\nu) + N_2 A_{21} + N_3 A_{31} - N_1 A_{10}$
- $dN_0/dt = (-N_0 B_{03} + N_3 B_{30}) \rho_P(\nu) + N_3 A_{30} + N_2 A_{20} + N_1 A_{10}$
- $d(N_0+N_1+N_2+N_3)/dt=0$
- *If $A_{32}$ greater than $A_{30}$ and $B_{30}\,\rho_P(\nu)$, as soon as $N_3 \neq 0$, then $dN_2/dt = N_3 A_{32}$ and $N_2 \neq 0$*
- *there is no gain since $N_2<N_1$ because $N_1$ related to temperature*
- *In order to sustain the gain cycle, the pump intensity must be greater than a threshold value.*
- **Ytterbium ion $Yb^{3+}$ can be pumped either 1-6 or 1-5**



# *Amplification : intensity and/or fluence*

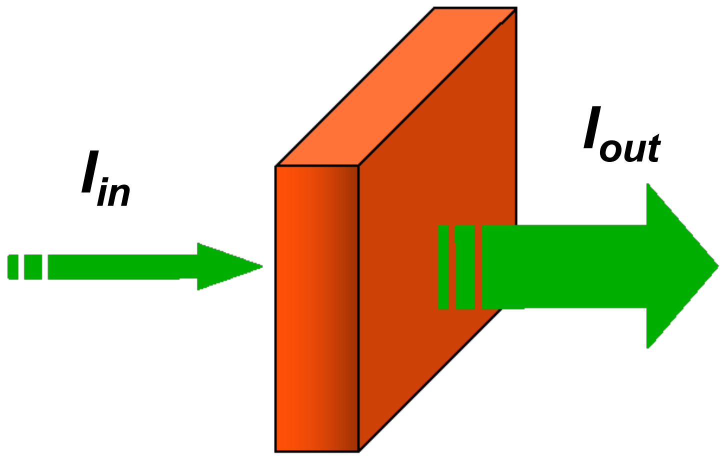
- *There is gain « g or $\gamma = \sigma\Delta n$»*

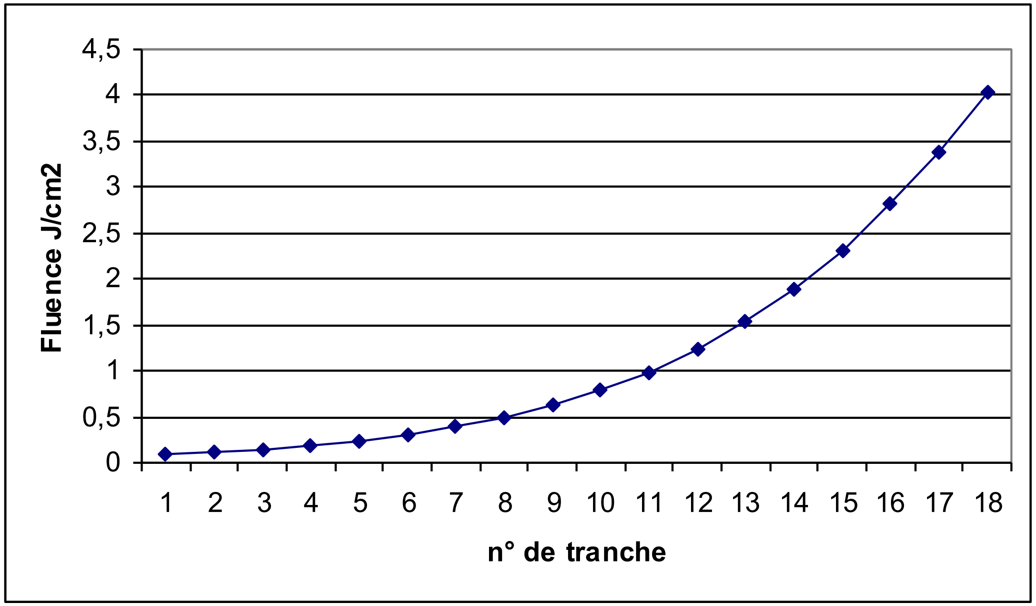
***Intensity :***

$$I_{out} = I_{in}\, Exp(g.l)$$

***Fluence***

$$F_{out} = F_{in}\, Exp(g.l)$$

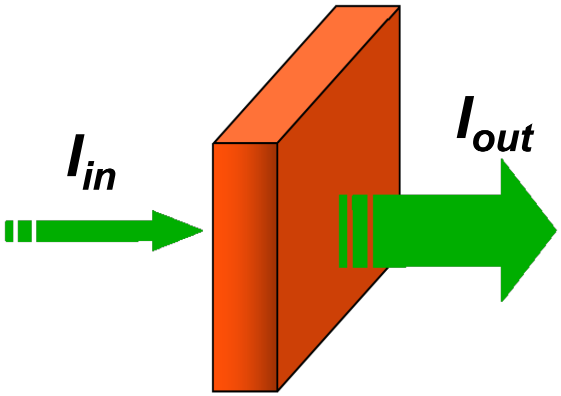


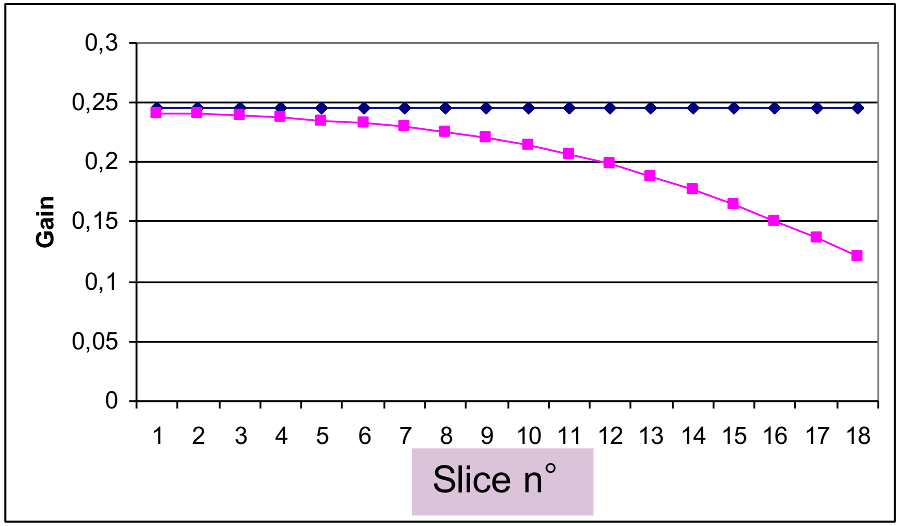




***What's going on if the thickness increases indefinitely ?***

# Amplification : intensity and/or fluence

*The medium is split in «n» slices (or «n» amplifiers are lined)*
*The blue curve is the initial gain, the rose curve is the gain after one pass*
*Since 1 photon is emitted ⇔ the atom "goes" from E2 to E1*
***and because** $\gamma = \sigma\Delta n$, the extracted energy is $\Delta E = \Delta N\, h\nu$*
*The more the amplification, the more the gain decreases*
*Same phenomenon on the temporal side*





*L. Frantz & J. Nodvik : « Theory of pulse propagation in a laser amplifier », J. Appl. Phys. 34,8, 2346 (1963)*

# Rate equations from Frantz and Nodvik*

- Starting from Maxwell equations, one gets a propagation equation «Helmoltz type» and a set of population equations (one equation per level) that can be reduced to:

$$\frac{\partial I}{\partial z} = \sigma I \Delta N \text{ et } \frac{\partial \Delta N}{\partial t} = -\frac{2\sigma I}{h\nu}\Delta N$$

- This is the Frantz et Nodvik model that is a function of one single «z» coordinate, and a function of time «t»
- This system has no analytical solution
- This system can be integrated formally as a function of time to lead to an energy relation (or fluence) in which the saturation fluence appears

$F_{sat} = h\nu/\sigma$

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**Theory of Pulse Propagation in a Laser Amplifier**

LEE M. FRANTZ AND JOHN S. NODVIK*
*Space Technology Laboratories, Inc., Redondo Beach, California*

(Received 7 March 1963)

# *Frantz and Nodvik*

Starting from the integrated formula (F is the fluence = energy E / surface ΔS) :

$$F_{out} = F_{sat} Ln(1 + Exp(gl)[Exp(F_{in}/F_{sat}) - 1])$$

It is straightforward to show that :

$$Exp(F_{out}/F_{sat}) - 1 = Exp(gl)[Exp(F_{in}/F_{sat}) - 1]$$

with

$$F_{in} << F_{sat} : F_{out}/F_{in} = Exp(gl)$$

$$F_{in} >> F_{sat} : F_{out} - F_{in} = glF_{sat} \text{ ou } \Delta E = glF_{sat}\Delta S$$

For a given amplification slice, one computes the residual gain after amplification:

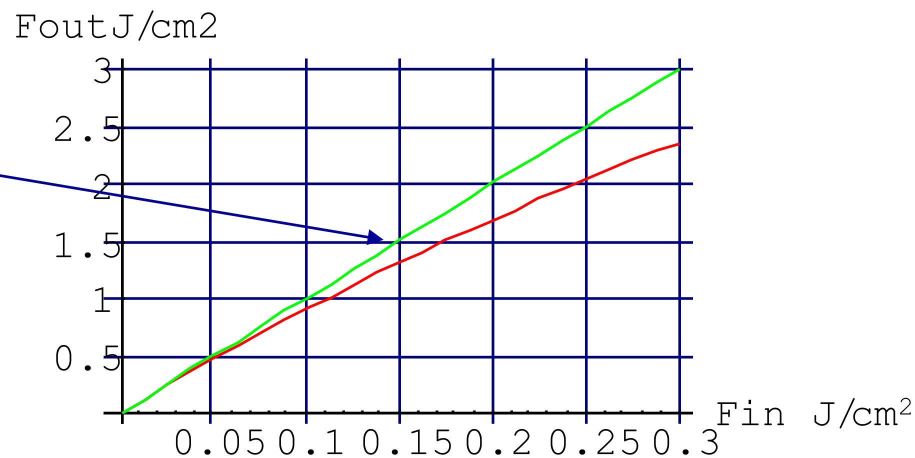
$$g_i l = -Ln[1 - (Exp(-F_{in(i-1)}/F_{sat}))(1 - Exp(-g_{i-1}l))]$$

An EXCEL file can be made easily.

*L. Frantz & J. Nodvik : « Theory of pulse propagation in a laser amplifier », J. Appl. Phys. 34,8, 2346 (1963)*

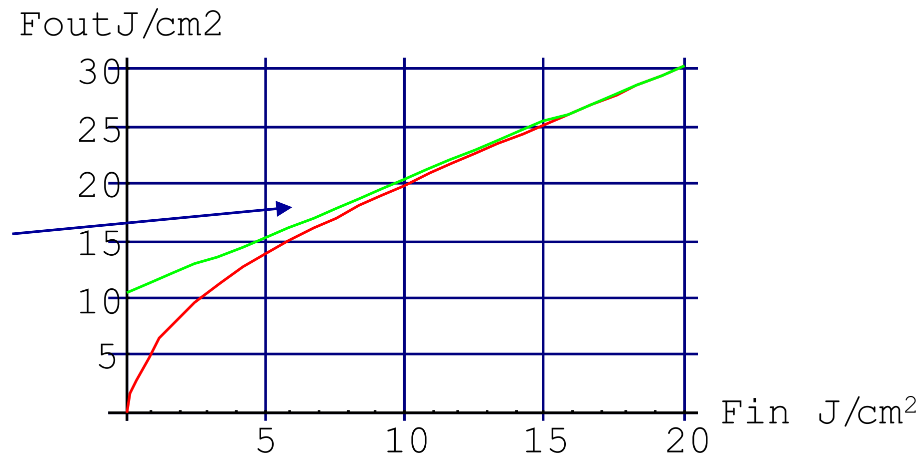
# *Frantz et Nodvik : 2 rates or regimes*

Linear behavior when $F_{in}$ <<$F_{sat}$, then $F_{out}/F_{in}$ = Exp(g*l)




*Example :*

- *l = 5cm,*
- *g = 0,0461 cm⁻¹*
- *(G = 10)*
- *$F_{sat}$ = 4,5 J/cm²*

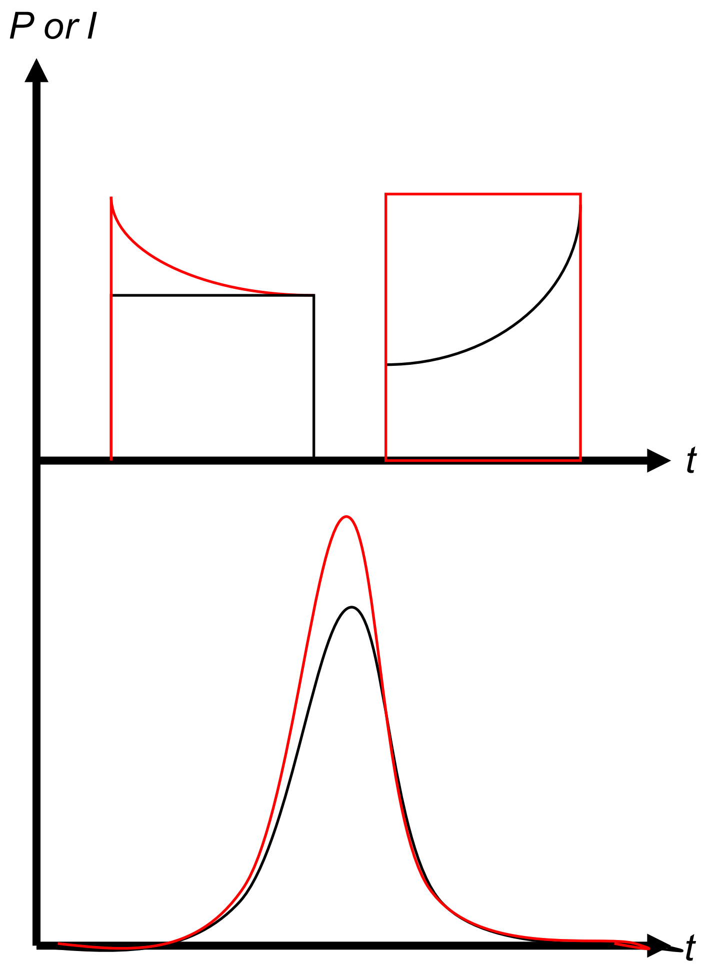



Saturated behavior when $F_{in}$ >>$F_{sat}$, then $F_{out}=F_{in}$ + g*l* $F_{sat}$

*Green curve approximated solution, red curve exact solution.*

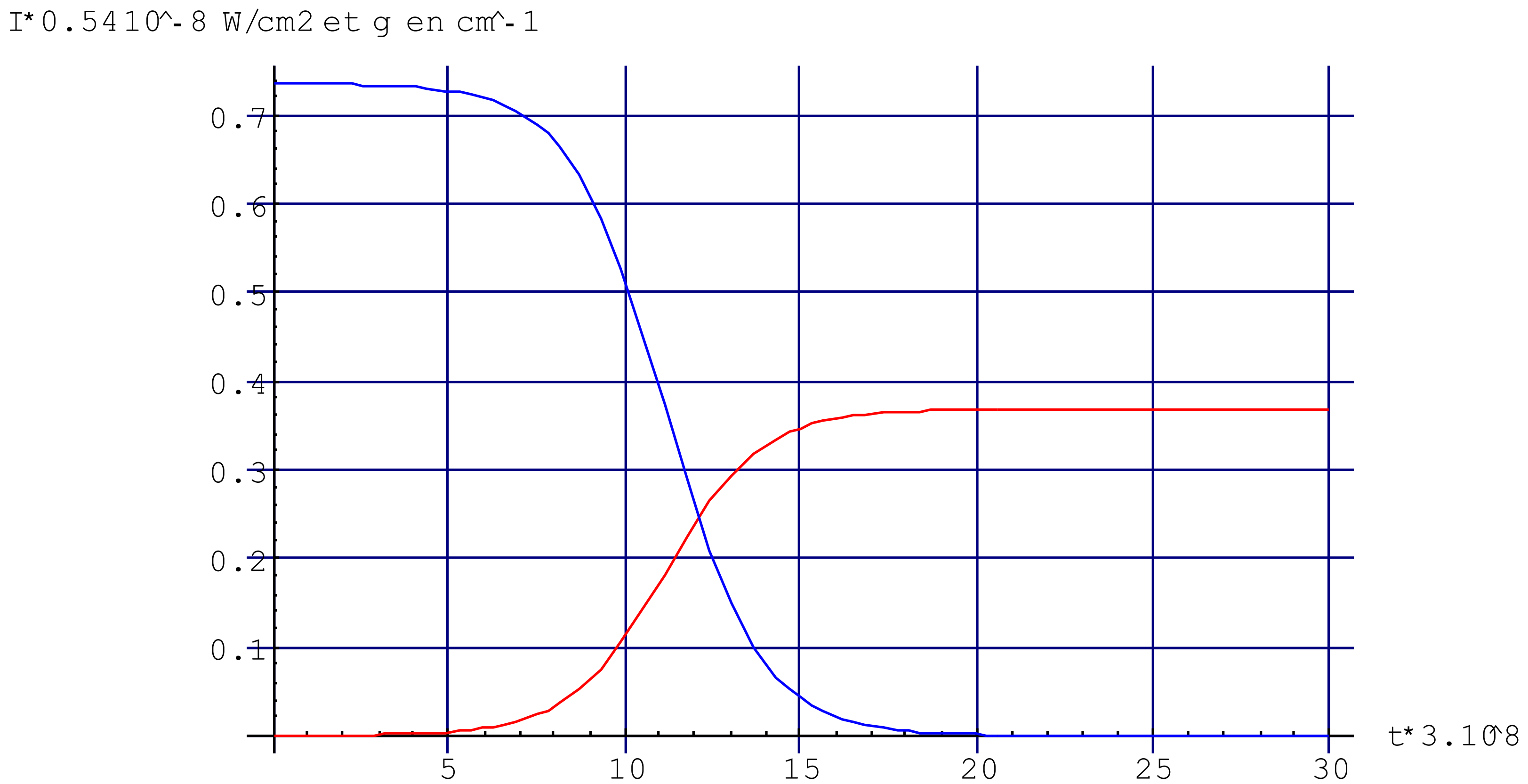
# *Temporal variations (1)*

- At a given z coordinate, for a thin slide of medium, at t = 0, the gain equals $g_0$.
- At any time t > 0, the gain is smaller than $g_0$ because I have used some part of the population inversion
- Therefore a square input pulse will be changed into a decreasing exponential shape
- Inversely, for a given square output pulse, I have to generate an increasing exponential input
- In the case of a Lorentzian or a Gaussian shape, the rising edge is more amplified than the leading edge. It seems that the pulse is going forward steeper and steeper



# Exact solution : an amplifier without loss

- Exact solution from Mathematica
- Gain = blue curve and intensity = red curve



$R_1$, $R_2$ are the intensity reflexion coefficients of the mirrors

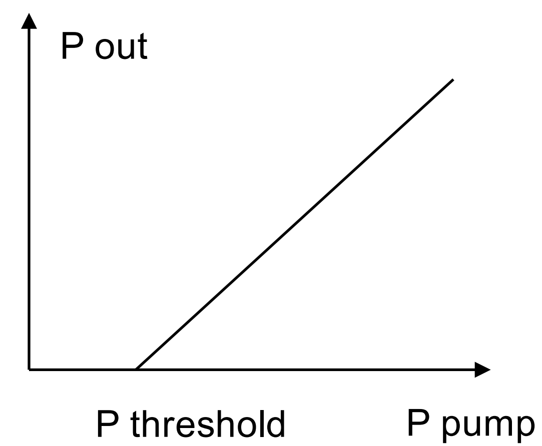
$g$ is the small signal gain

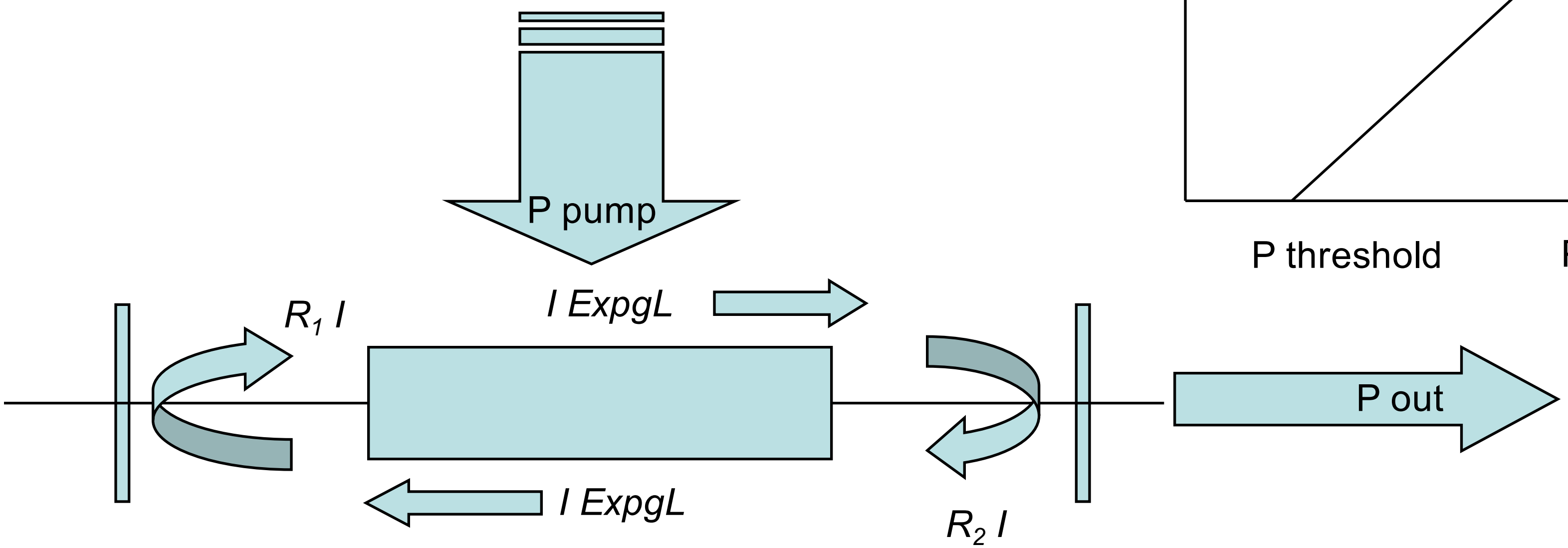
$\alpha$ are the losses, $L_c$ the cavity length, $L$ the gain medium length

The oscillation condition is *gain = losses* on a single round-trip:

$R_1 R_2 \, Exp\,(2gL-2\alpha L_c)=1$

Linear behaviour above threshold

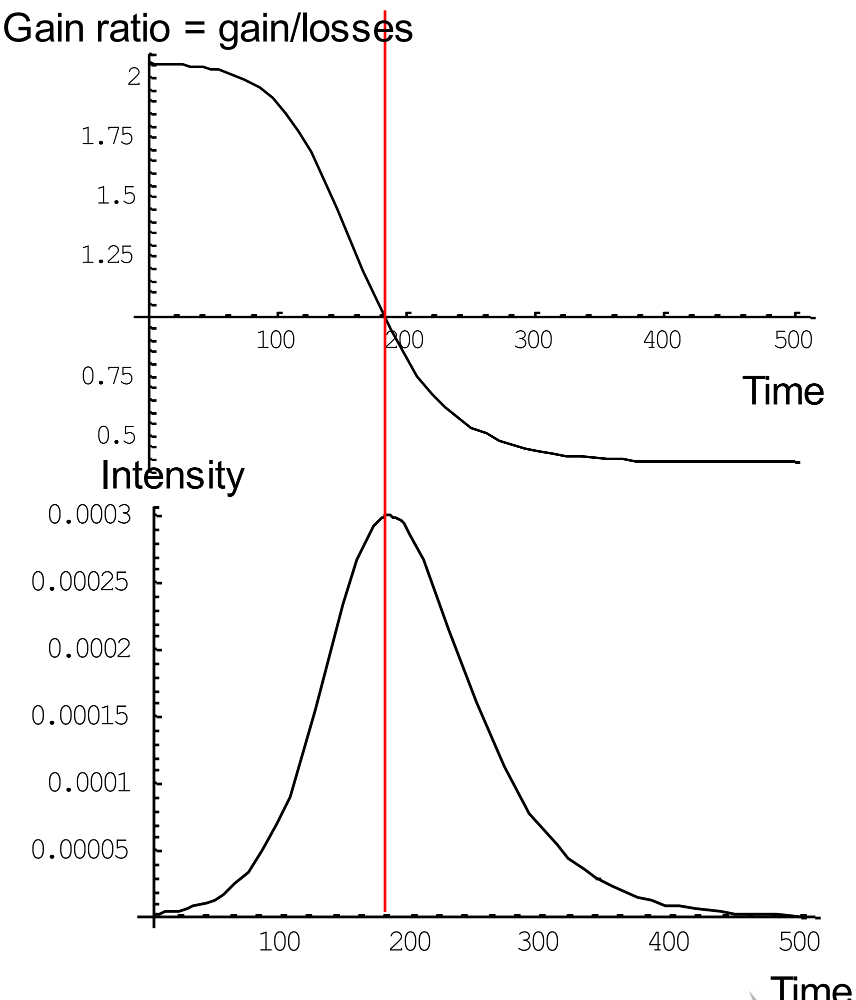


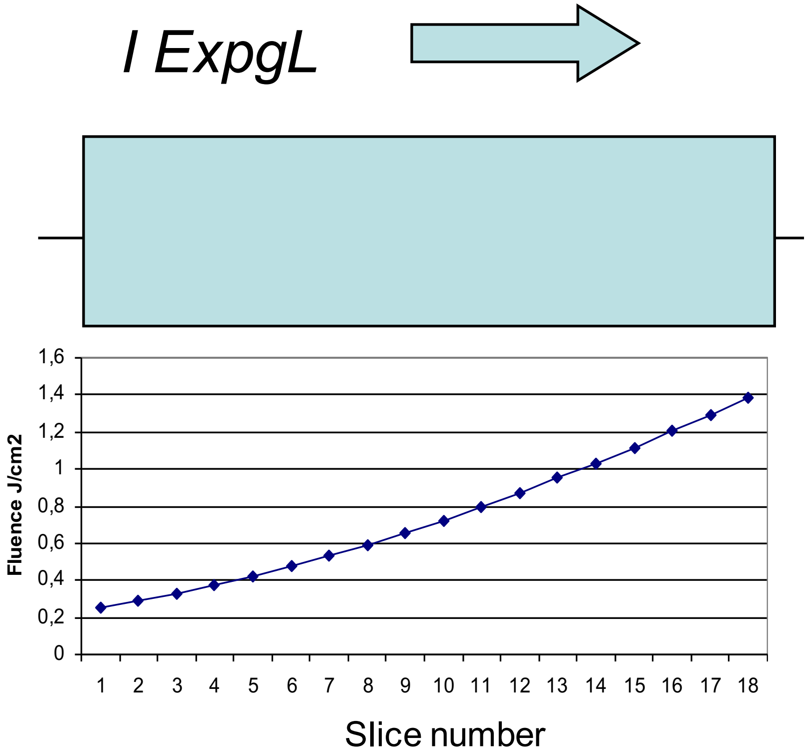
# *Exact solution: cavity with losses*

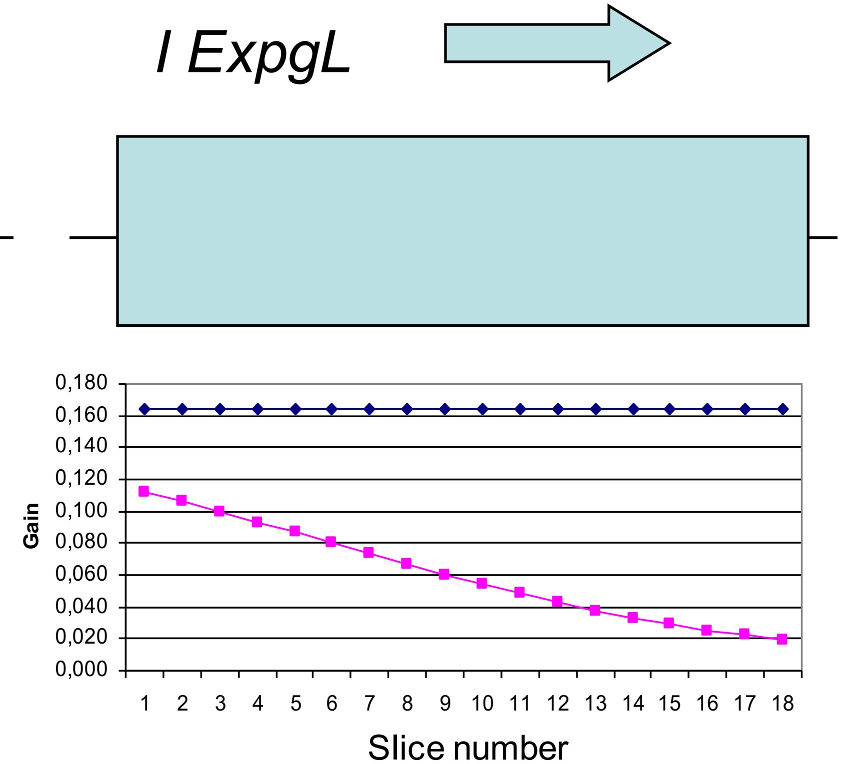
- Cavity losses considered as a threshold for gain : $R_1 R_2$ Exp $(2gL-2\alpha L_c)=1$

- When this gain ratio equals or is greater than1, intensity increases until gain ratio goes back to 1, then intensity decreases



# Single pass amplification

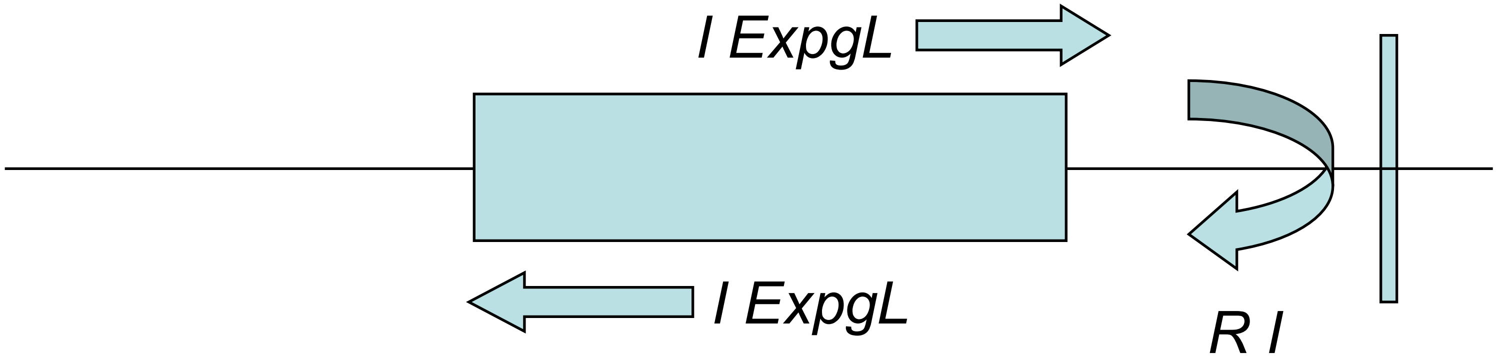


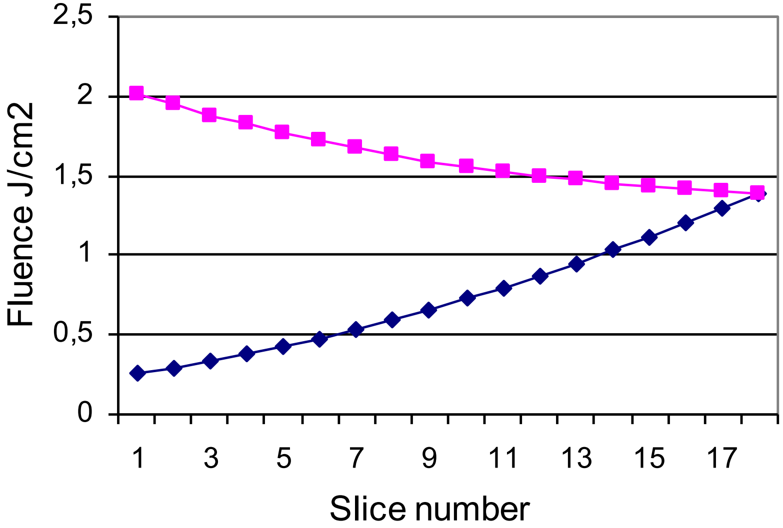



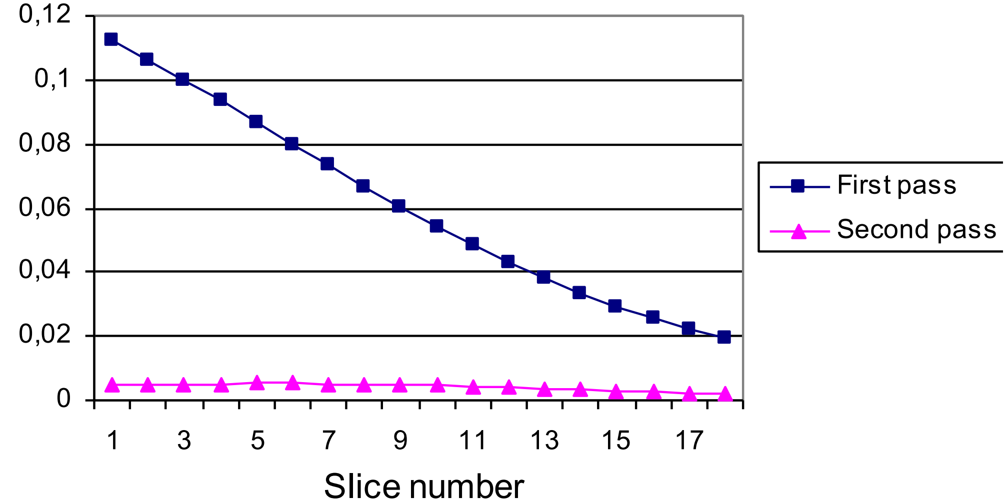

$$F_{out}=F_{sat}Ln(1+Exp(gl)[Exp(F_{in}/F_{sat})-1])$$

$$g_{i}l=-Ln[1-(Exp(-F_{in(i-1)}/F_{sat}))(1-Exp(-g_{i-1}l))]$$

# Double pass amplification

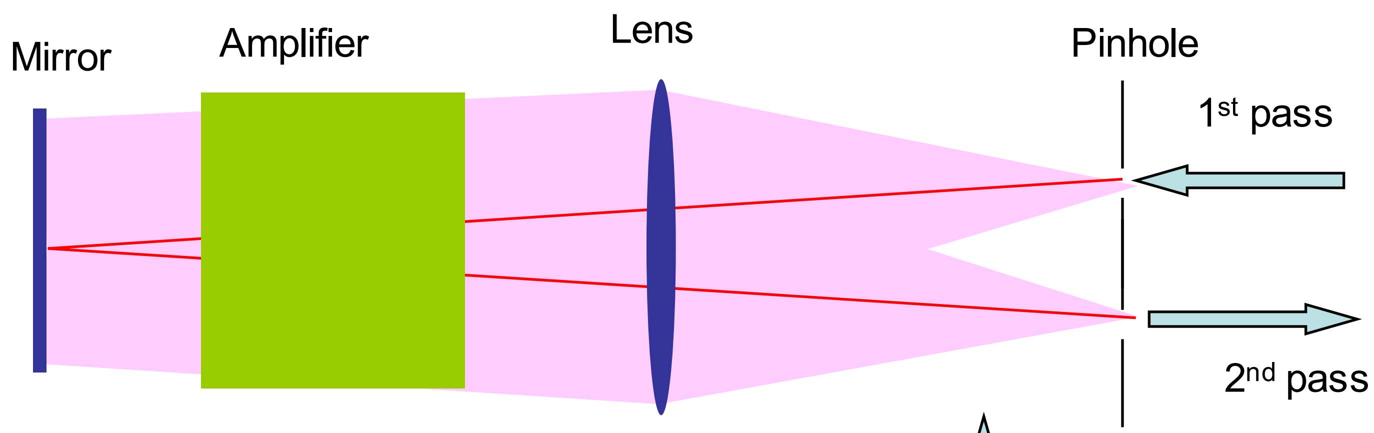


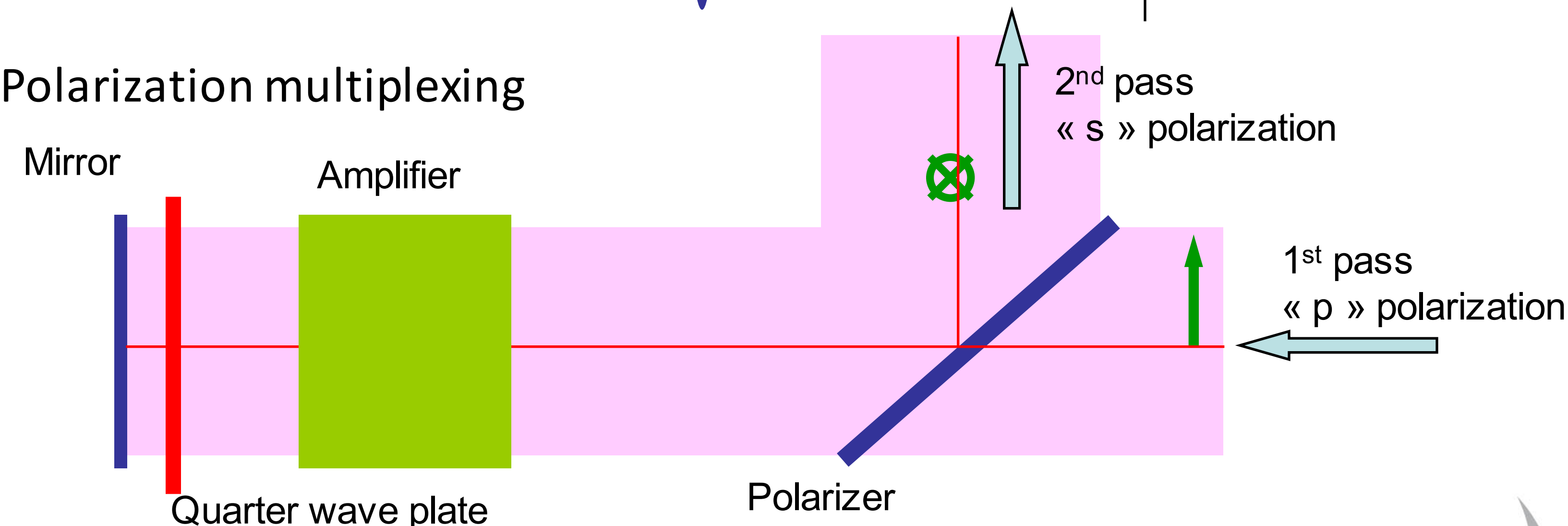



# Double pass amplifier

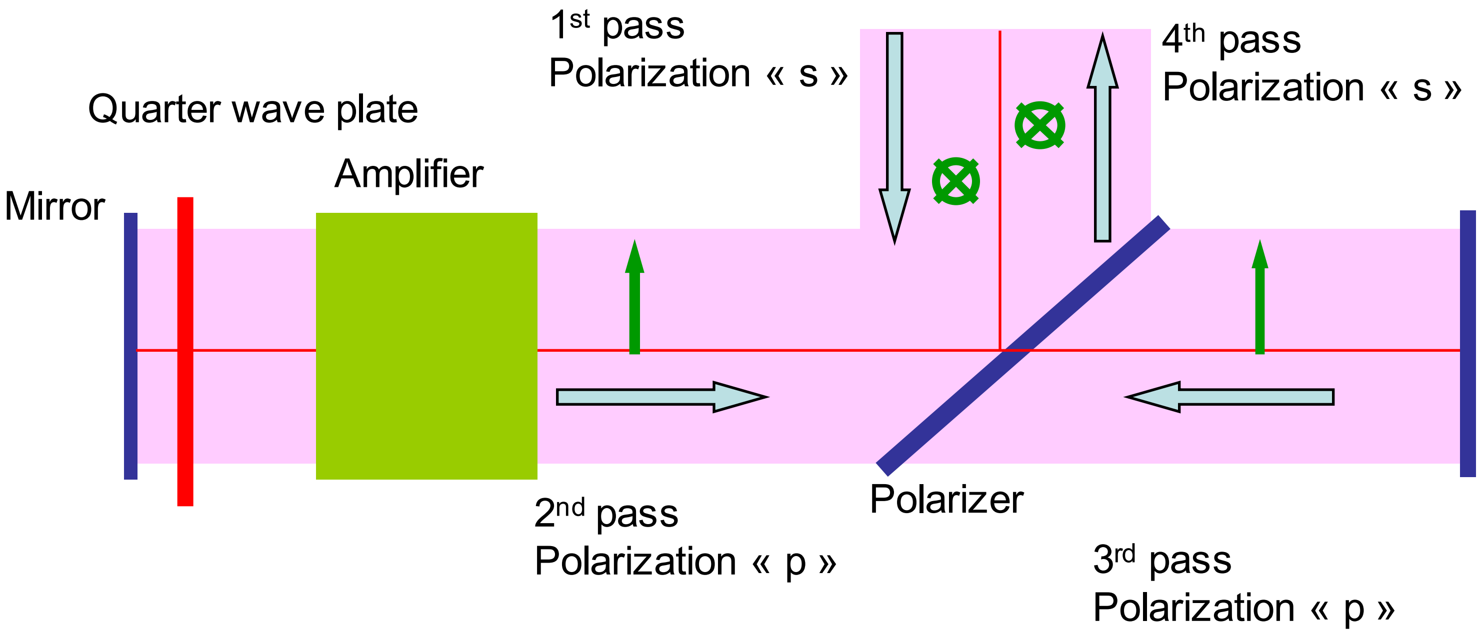
- Angular multiplexing




- Polarization multiplexing

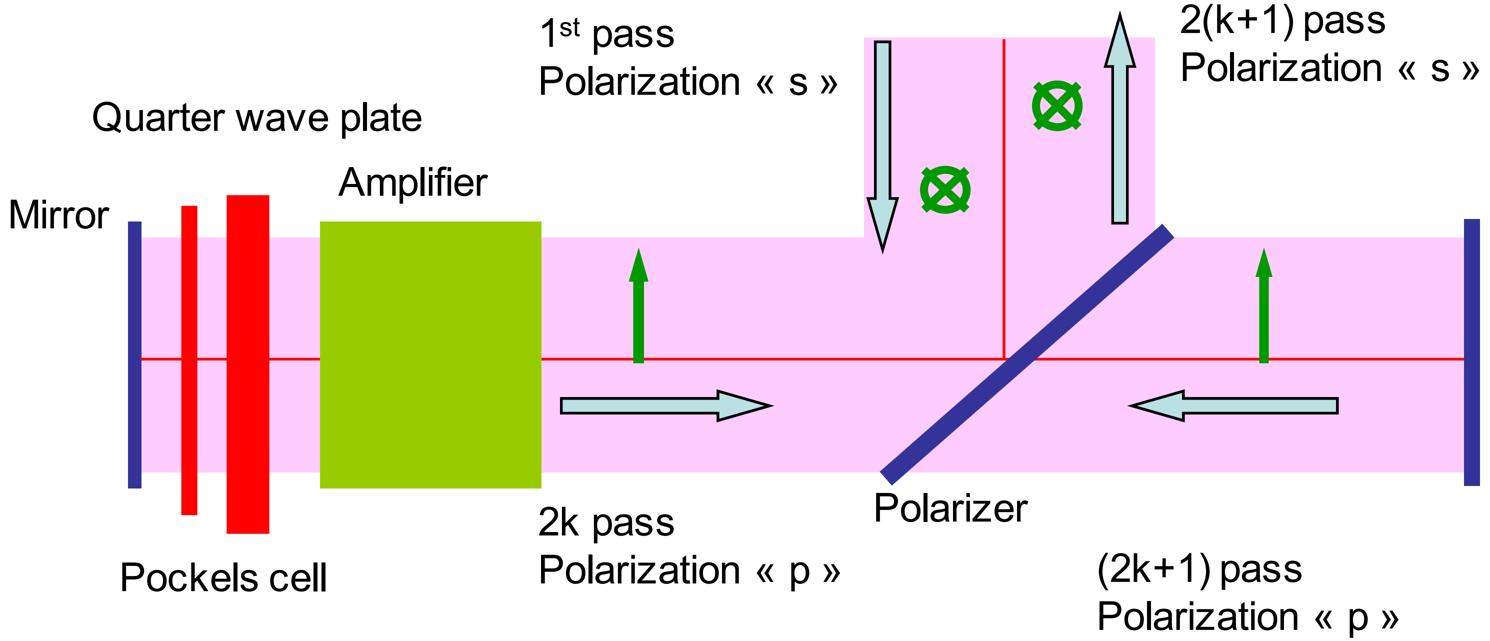


# 4 pass amplifier



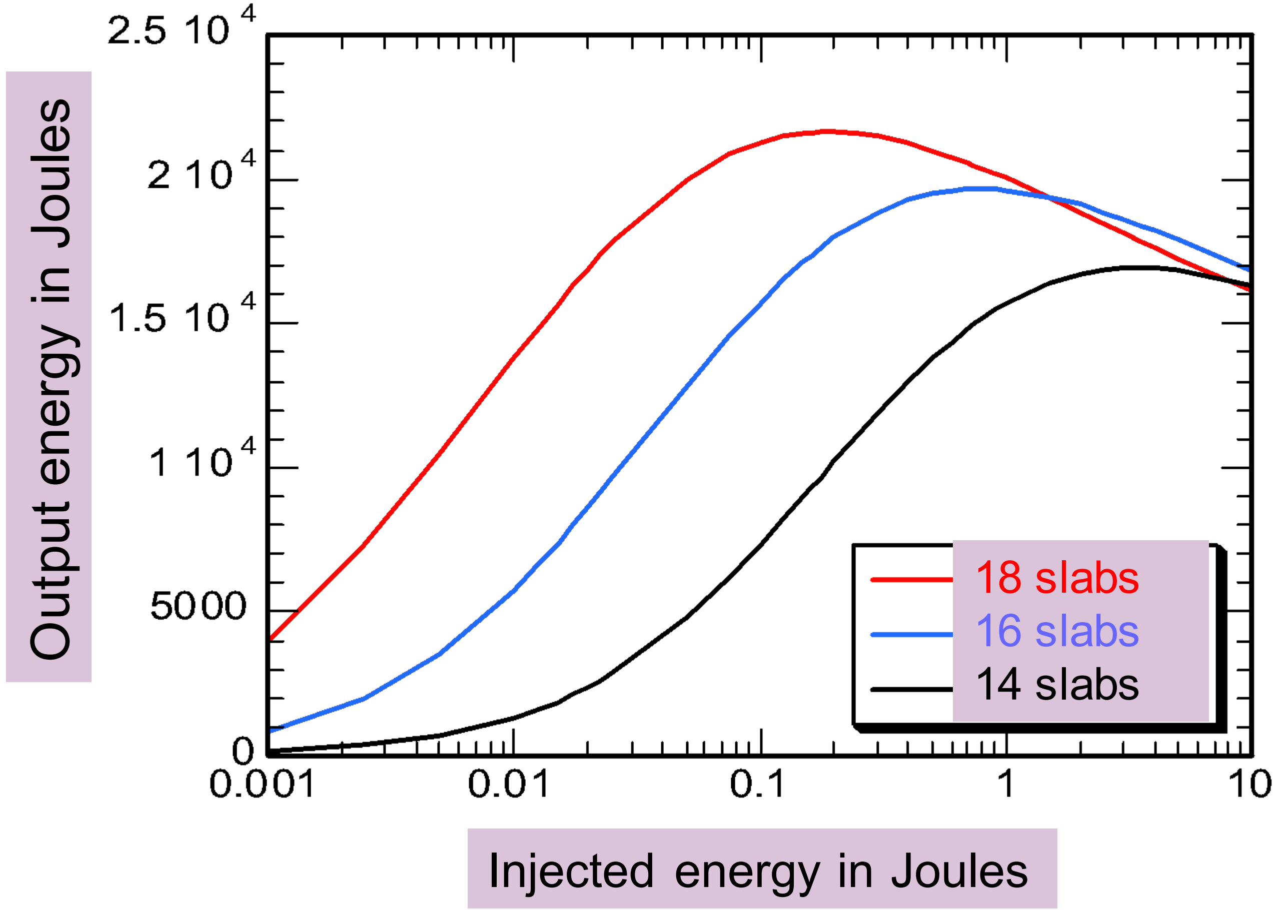

- Quarter wave plate = 4 pass
- Pockels cell to switch « on » and « off »

# n pass amplifier




- Phase difference: $\pi/2$ ($\lambda/4$) between the 2nd and the 3rd pass
- Output possible at the 2(k+1) pass if the phase difference $-\pi/2$ ($-\lambda/4$) between the 2k and the (2k+1) pass

## Optimization : 1$\omega$ output energy as a function of injected energy in a four pass amplifier with 14, 16 or 18 slabs



# End of part 1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