Advanced Summer School on
"Laser-Driven Sources of High Energy Particles and Radiation"
9-16 July 2017, Capri, Italy

Consiglio Nazionale
delle Ricerche

INO-CNR
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Fisica Nucleare

Ultrafast, intense laser pulse diagnostics
(Lecture 1 of 2)

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The Intense Laser Irradiation Laboratory (ILIL) group

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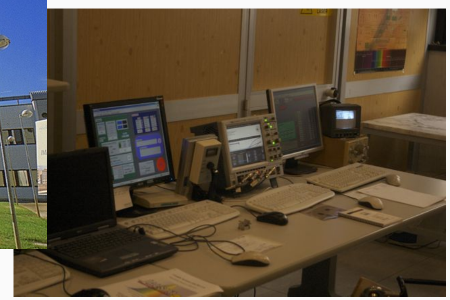

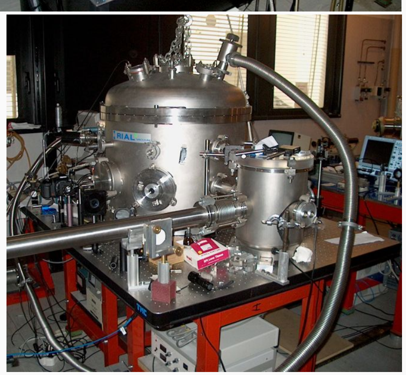

PEOPLE

- Leonida A. GIZZI (CNR)*, scientist (lab resp.)
- Giancarlo BUSSOLINO (CNR), scientist
- Gabriele CRISTOFORETTI (CNR), scientist
- Luca LABATE (CNR)*, scientist
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- Antonio GIULIETTI(CNR), associated scientist
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- Antonella ROSSI (CNR) – Tech.

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Main research topics

Laser-wakefield acceleration
- Self-injection mechanisms
- X-ray and γ-ray generation
- Radiobiology with laser-driven electrons

Ultraintense laser-solid interactions
- Protons/light ion acceleration
- X-ray diagnostic development
- Laser-plasma interaction characterization

Laser development

For further infos: http://ilil.ino.it

www.ino.it

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Summary of ILIL laser performances (upgrade Q3/2013)

Laser in figures

- energy: up to 450mJ on target
- pulse duration <40fs
- ASE contrast > 10^9
- final beam diameter ~43mm
- $M^2 < 1.5$
- rep rate: 10Hz

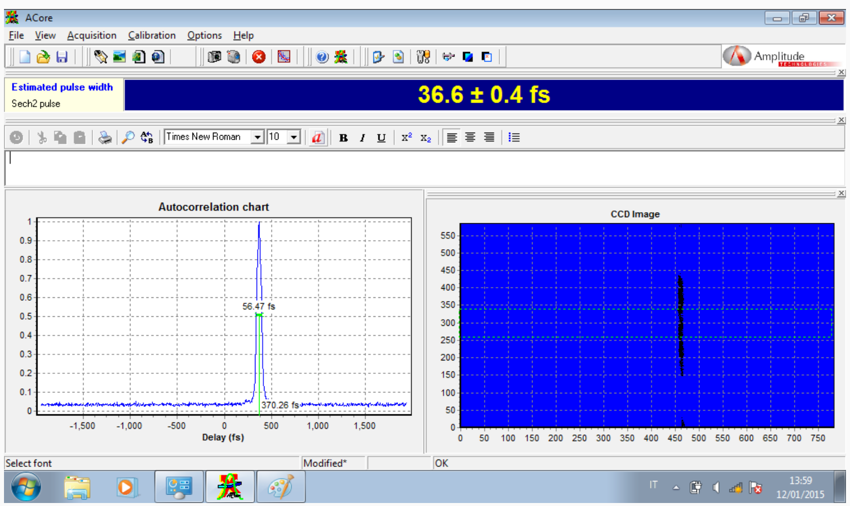
Focusing optics

Long focal length OAP

f/#: 11.5
W: 16.6 µm
I_{max}*: 2x10*^{18} *W/cm*^{2}
Z_R*: 620 µm*
("LWFA" experiments)

Short focal length OAP

f/#: 3.5
W: 5 µm
I_{max}*: 2x10*^{19} *W/cm*^{2}
Z_R*: 56 µm*
("TNSA" experiments)



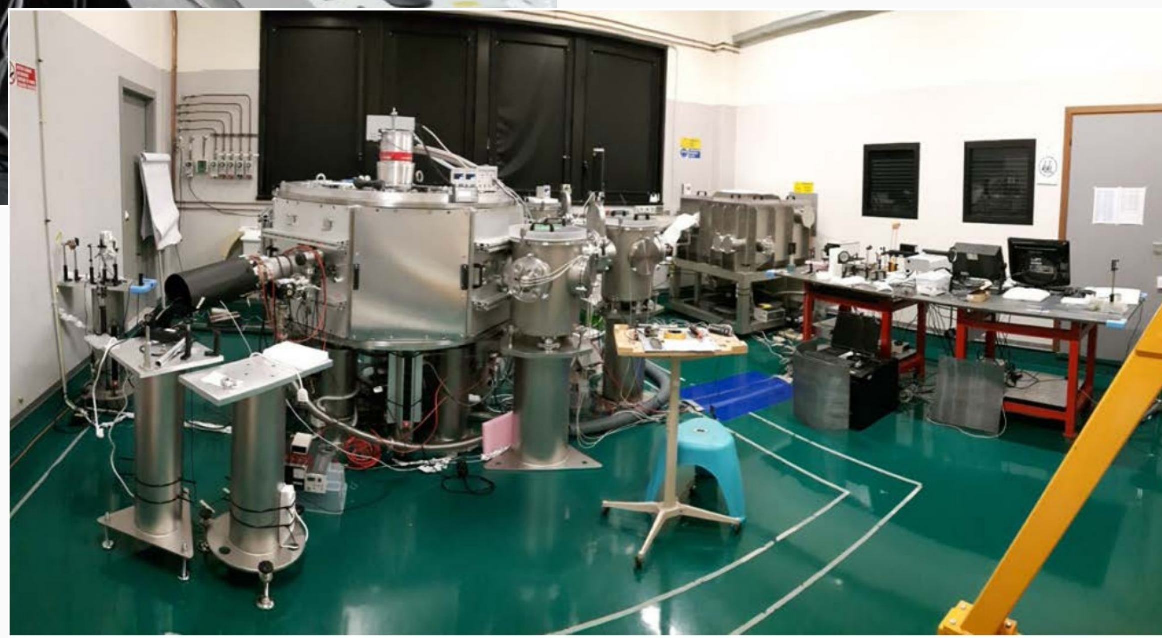
2^{nd} order autocorrelation

ILILsubPW upgrade 2016-2017

Laser in figures

- energy: up to 3J on target
- pulse duration 30fs
- 100TW power
- ASE contrast $> 10^9$
- rep rate: 1Hz

Final amplifier room



New Target Area (before radioprotection bunker construction)

Outline

Lecture 1 of 2

A (not so short) introduction to the mathematical description of the temporal behaviour of ultrashort laser pulses (terminology,, basic facts, ...)

- Spectral amplitude and phase
- Dispersion, dispersion compensation

Experimental techniques for the temporal characterization of ultrashort laser pulses

- Photodiodes, streak camera
- 1st and 2nd order autocorrelators

Lecture 2 of 2

- Advanced techniques for the pulse length and spectral phase measurements: FROG, SPIDER
- Contrast measurement techniques (in brief)

Transverse functions characterization and wavefront correction

- Wavefront characterization techniques
- Wavefront correction and beam focusing

Mathematical description of the temporal behaviour of ultrashort pulses: basic facts

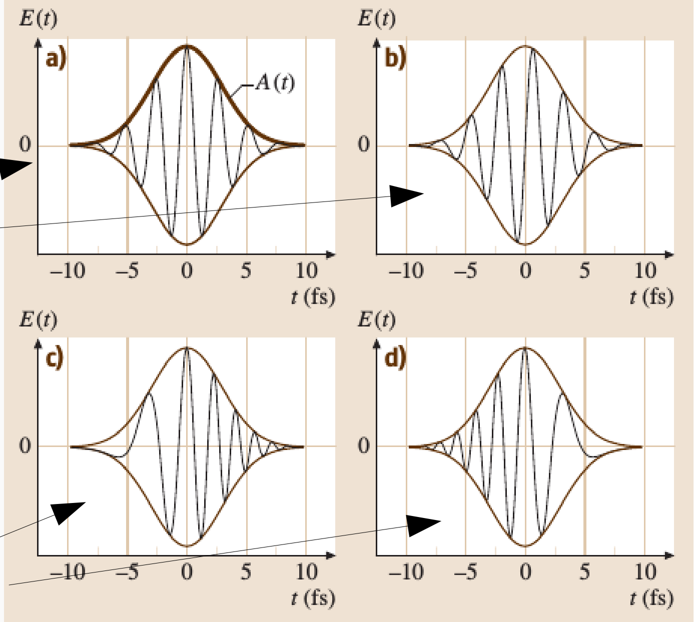
At a fixed point in space, for a linearly polarized pulse, the electric field can be simply written as $E(t) = A(t)\cos(\Phi_0 + \omega_0 t)$

The field envelope and the pulse intensity are related by the expression (A in V/m, I in W/cm2)

$$A(t) = \sqrt{\frac{2}{\varepsilon_0 c}}\sqrt{I(t)} = 27.4\sqrt{I(t)}$$

Φ_0 Absolute phase,
or *carrier envelope phase (CEP)*

Cosine pulse
Sine pulse



Adding a time dependent phase function results in a so-called *chirped* pulse

$$\Phi(t) = \Phi_0 + \omega_0 t + \Phi_a(t)$$

An *instantenous frequency* can be defined as

$$\omega(t) = \frac{d\Phi(t)}{dt} = \omega_0 + \frac{d\Phi_a(t)}{dt}$$

Upchirped pulse
Downchirped pulse

As we will see, there is no way to directly characterize the pulse duration (and phase) in the time domain (that is, to "directly" measure, for instance, *E(t)*), so that we need to think also to the frequency domain for a complete description

Mathematical description of the temporal behaviour of ultrashort pulses: Introducing the spectral amplitude and phase

Using Fourier analysis, the field and its Fourier transform can be written as

$$E(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{E}(\omega)\,\mathrm{e}^{\mathrm{i}\omega t}\,\mathrm{d}\omega \qquad \tilde{E}(\omega) = \int_{-\infty}^{\infty} E(t)\,\mathrm{e}^{-\mathrm{i}\omega t}\,\mathrm{d}t$$

Of course, the knowledge of one of the two description is enough to completely characterize the pulse.

Most often, the so called "analytic signal" is used.

Being *E(t)* real, its Fourier transform is a Hermitian function: $\tilde{E}(\omega) = \tilde{E}^*(-\omega)$

This means that the knowledge of the Fourier transform for positive frequencies is enough to fully retrieve the signal

We can thus *define*, for convenience, a new function in the frequency domain, retaining only the positive part of the FT:

$$\tilde{E}^+(\omega) = \begin{cases} \tilde{E}(\omega) & \text{for } \omega \geq 0 \\ 0 & \text{for } \omega < 0 \end{cases}$$

and the corresponding FT^{-1} $\quad E^+(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{E}^+(\omega)\,\mathrm{e}^{\mathrm{i}\omega t}\,\mathrm{d}\omega$

See for instance G.J. Gbur, *Mathematical Methods for Optical Physics and Engineering,* Cambridge University Press 2011

Mathematical description of the temporal behaviour of ultrashort pulses: Introducing the spectral amplitude and phase

According to the above observation, $E^+(t)$, which is called *analytic signal**, is enough to retrieve the "real" field $E(t)$

Indeed, it can be easily demonstrated that $E(t) = 2\,\mathrm{Re}\{E^+(t)\}$

(Hint: define a similar function $E^-(\omega)$ in the frequency domain, only retaining the negative spectral part, identify the FT of $E(t)$ as the sum of E^+ and E^-, transform back to the time domain, ...)

$E^+(t)$ is a complex function, so that it can be written as

$$E^+(t) = |E^+(t)|e^{i\Phi(t)} = |E^+(t)|e^{i\Phi_0}e^{i\omega_0 t}e^{i\Phi_a(t)} \propto \sqrt{I(t)}e^{i\Phi_0}e^{i\omega_0 t}e^{i\Phi_a(t)} \propto A(t)e^{i\Phi_0}e^{i\omega_0 t}e^{i\Phi_a(t)}$$

where the meaning of the different parameters is the same as above

We will most often use the analytic signal in the following (sometimes, the + sign will be omitted for convenience)

*see L. Mandel, E. Wolf, *Optical Coherence and Quantum Optics* for an explanation

Mathematical description of the temporal behaviour of ultrashort pulses: Introducing the spectral amplitude and phase

Similarly, in the spectral domain we can introduce a *spectral amplitude* and a *spectral phase* as

$$\tilde{E}^{+}(\omega) = |\tilde{E}^{+}(\omega)|e^{-i\phi(\omega)} \propto \sqrt{I(\omega)}e^{-i\phi(\omega)}$$

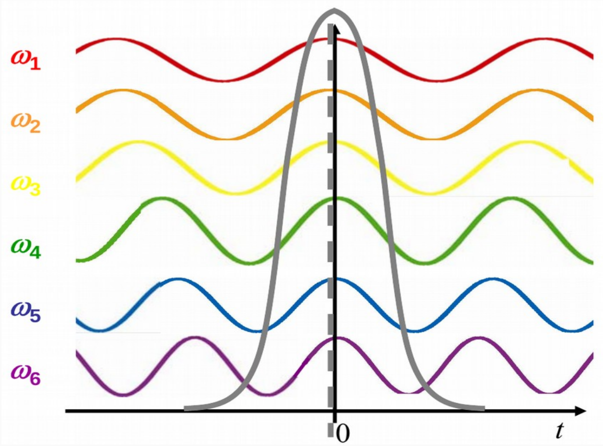
What do the spectral amplitude and phase mean?

Spectral amplitude: proportional to the square root of *I(ω)*, the usual "spectrum" as measured by a spectrometer

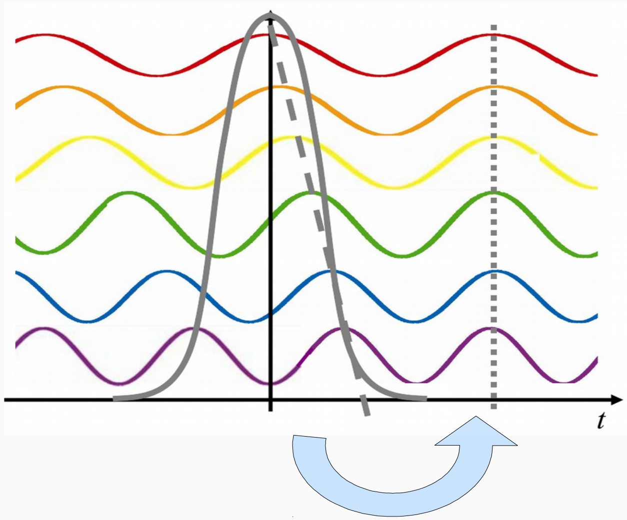
The spectral phase is basically the phase of each frequency in the waveform

What is the effect in the time domain? Two examples

$$\phi(\omega_i) = 0$$



$$\phi(\omega_i) = (i-1)\frac{\pi}{5}$$



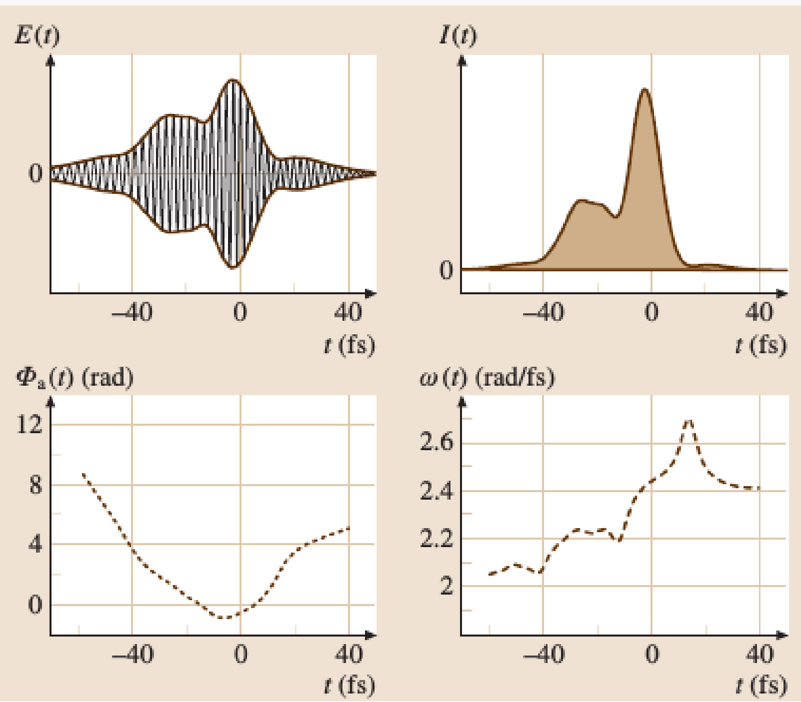
Mathematical description of the temporal behaviour of ultrashort pulses: Time vs frequency domain descriptions

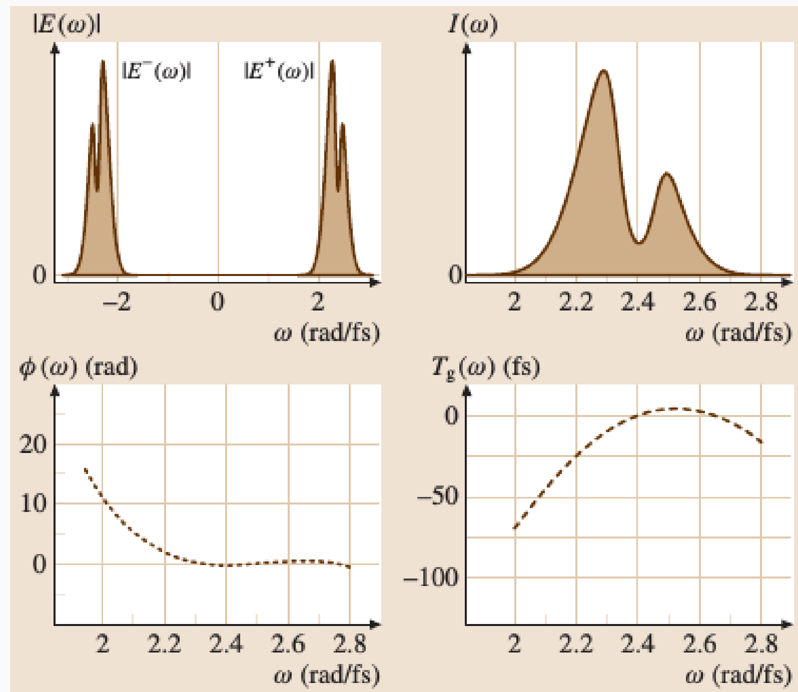
Time domain

$$E(t) = A(t)e^{i\Phi_0}e^{i\omega_0 t}e^{i\Phi_a(t)}$$

Frequency domain

$$\tilde{E}^+(\omega) = |\tilde{E}^+(\omega)|e^{-i\phi(\omega)} \propto \sqrt{I(\omega)}e^{-i\phi(\omega)}$$





Mathematical description of the temporal behaviour of ultrashort pulses: What does the spectral phase mean?

Actually, in the two examples two slides above we considered a linear dependence upon ω

In general, the spectral phase can be expanded into a Taylor series:

$$\phi(\omega) = \sum_{j=0}^{\infty} \frac{\phi^{(j)}(\omega_0)}{j!} \cdot (\omega - \omega_0)^j \qquad \text{where, of course,} \qquad \phi^{(j)}(\omega_0) = \left.\frac{\partial^j \phi(\omega)}{\partial \omega^j}\right|_{\omega_0}$$

This holds for a *well-defined* pulse. Basically, it means that each term in the expansion produces a pulse broadening or distortion that is significantly smaller than that of the previous term (see * for a deeper discussion on the optical meaning)

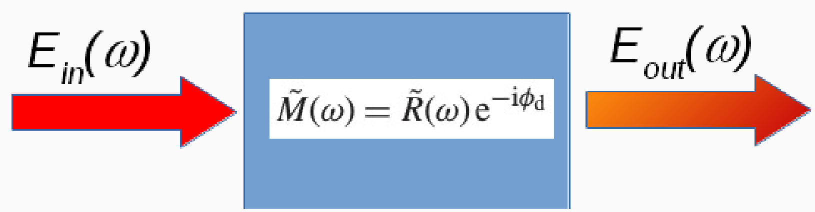
$$\left.\frac{\partial \phi}{\partial \omega}\right|_{\omega_0} (\omega - \omega_0) \gg \frac{1}{2!}\left.\frac{\partial^2 \phi}{\partial \omega^2}\right|_{\omega_0} (\omega - \omega_0)^2 \gg \frac{1}{3!}\left.\frac{\partial^2 \phi}{\partial \omega^2}\right|_{\omega_0} (\omega - \omega_0)^3 \gg \ldots$$

Terminology: 2nd order term → Group Velocity Dispersion (GVD), 3rd order term → Third Order Dispersion (TOD)

Why introducing the spectral amplitude and phase?

A linear optical system acts on an input field by a multiplication by a (complex) transfer function in the frequency domain:

$$\tilde{E}^{+}_{\text{out}}(\omega) = \tilde{M}(\omega)\tilde{E}^{+}_{\text{in}}(\omega) = \tilde{R}(\omega)\mathrm{e}^{-\mathrm{i}\phi_{\mathrm{d}}}\tilde{E}^{+}_{\text{in}}(\omega)$$



The spectral phase of the output pulse is thus modified according to

$$\phi_{in}(\omega) \mapsto \phi_{in}(\omega) + \phi_d(\omega)$$

An initially unchirped pulse ($\phi''_{in}(\omega_0) = 0$) can acquire a chirp if $\phi''_d(\omega_0) \neq 0$ (more details in a moment)

*see D.N. Fittinghoff *et al.*, IEEE J. Sel. Top. Quant. Electr. **4**, 430 (1998)

Spectral phase: the meaning of the first orders

Spectral phase expansion $$\phi(\omega)=\sum_{j=0}^{+\infty}\frac{1}{j!}\phi^{(j)}(\omega_0)(\omega-\omega_0)^j$$

Reference pulse with $\phi(\omega)=0$, so that $\tilde{E}_{ref}(\omega)=\left|\tilde{E}_{ref}(\omega)\right|$ and $E^+_{ref}(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\left|\tilde{E}_{ref}(\omega)\right|\mathrm{e}^{i\omega t}\,\mathrm{d}\omega$

$\phi(\omega)=\phi(\omega_0)$ Pulse with a constant (zero order) term

$$\tilde{E}^+(\omega)=\left|\tilde{E}^+(\omega)\right|\mathrm{e}^{-i\phi(\omega_0)}=\left|\tilde{E}^+_{ref}(\omega)\right|\mathrm{e}^{-i\phi(\omega_0)}$$

On calculating the IFT, one gets

$$E^+(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\left|\tilde{E}^+(\omega)\right|\mathrm{e}^{-i\phi(\omega_0)}\mathrm{e}^{i\omega t}\,\mathrm{d}\omega=\mathrm{e}^{-i\phi(\omega_0)}E^+_{ref}(t)$$

This corresponds to acquiring an absolute phase $\phi(\omega_0)$

$\phi(\omega)=\phi'(\omega_0)(\omega-\omega_0)$ Pulse with a 1st order term

$$\tilde{E}^+(\omega)=\left|\tilde{E}_{ref}(\omega)\right|\mathrm{e}^{-i\phi'(\omega_0)(\omega-\omega_0)}$$

On calculating the IFT, one gets

$$E^+(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\left|\tilde{E}^+_{ref}(\omega)\right|\mathrm{e}^{-i\phi'(\omega_0)(\omega-\omega_0)}\mathrm{e}^{i\omega t}\,\mathrm{d}\omega=\mathrm{e}^{i\phi'(\omega_0)\omega_0}\tilde{E}^+(t-\phi'(\omega_0))$$

This corresponds to a time shift of the pulse, with

$$T_g=\phi'(\omega_0)$$

→ To summarize: constant and linear terms in the spectral phase have no effects on the pulse duration

Higher order terms and pulse duration

For a pulse with a given bandwidth (and spectrum), the shortest duration is reached when no chirp occurs; in the frequency domain, this translates into the spectral phase exhibiting a constant or linear dependence upon ω

$$\mathcal{F}\left[(t-\langle t\rangle)E(t)\right] = i\mathrm{e}^{-i\omega\langle t\rangle}\frac{\partial}{\partial\omega}\left(\mathrm{e}^{i\omega\langle t\rangle}\tilde{E}(\omega)\right)$$

(easy to demonstrate)

We start calculating the pulse duration for a general pulse as

$$\Delta t^2 = \int_{-\infty}^{+\infty}(t-\langle t\rangle)^2 I(t)\,\mathrm{d}t = \int_{-\infty}^{+\infty}\left|(t-\langle t\rangle)E(t)\right|^2\,\mathrm{d}t$$

Using the Plancherel's identity and the equation aside

$$= \int_{-\infty}^{+\infty}\left|\mathcal{F}\left[(t-\langle t\rangle)E(t)\right]\right|^2\,\mathrm{d}\omega = \int_{-\infty}^{+\infty}\left|\frac{\partial}{\partial\omega}\left(\mathrm{e}^{i\omega\langle t\rangle}\tilde{E}(\omega)\right)\right|^2\,\mathrm{d}\omega$$

and finally, on introducing the spectral amplitude and phase and calculating the derivative

$$\Delta t^2 = \int_{-\infty}^{+\infty}\left|\frac{\partial}{\partial\omega}\left|\tilde{E}(\omega)\right|\right|^2\,\mathrm{d}\omega + \int_{-\infty}^{+\infty}\left|\tilde{E}(\omega)\right|^2\left|\frac{\partial}{\partial\omega}\left(\omega\langle t\rangle-\phi(\omega)\right)\right|^2\,\mathrm{d}\omega$$

The first integral is ever positive and depends upon the spectral amplitude (or, the spectrum). As for the second one:

$$\frac{\partial}{\partial\omega}\left(\omega\langle t\rangle-\phi(\omega)\right) = \langle t\rangle-\phi'(\omega_0)-\frac{\partial}{\partial\omega}(\text{spectral phase terms } O((\omega-\omega_0)^2))$$

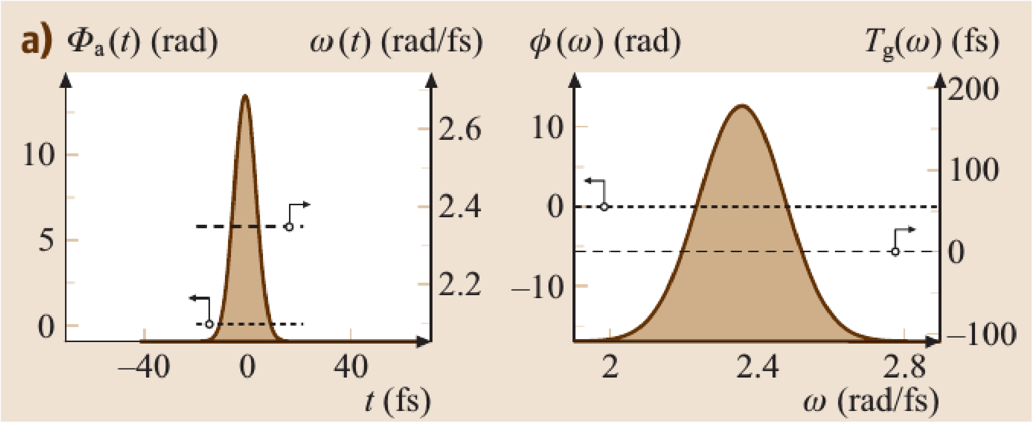
We saw above that the second term accounts, in the time domain, for a pulse delay, $T_g = \phi'(\omega_0)$ so that the first two terms cancels out

Thus, a further (positive) contribution to the time duration exists if the spectral phase exhibits higher order terms (GVD, TOD, ...)

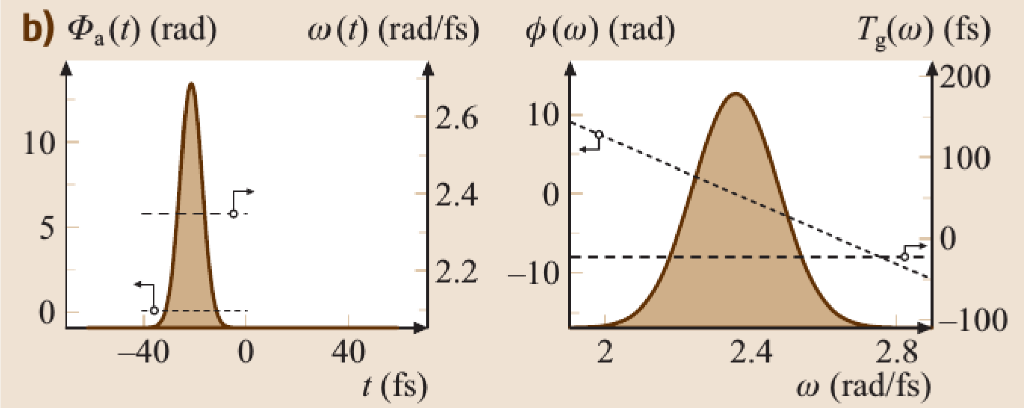
Ideally, in order to keep a pulse as short as possible, one thus look for optical elements which do not transfer quadratic phase to the pulse or (most of times!) for devices for adjusting/compensating for the accumulated spectral phase

See also I. Walmsley *et al.*, Rev. Sci. Instr. **72**, 1 (2001)

Spectral phase modifications and time duration: a few examples



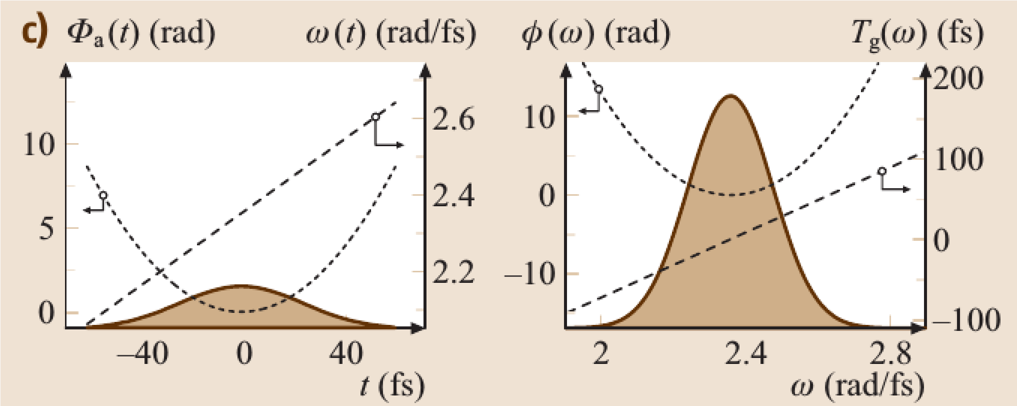
Unchirped (bandwidth limited) pulse, constant spectral phase



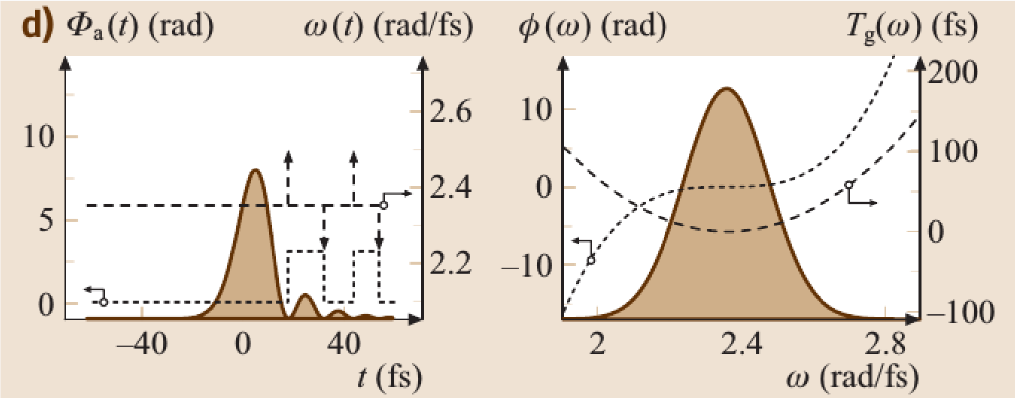
Unchirped pulse (bandwidth limited), shifted in time due to a linear (negative) spectral phase

Figure credits: Springer Handbook of Laser Optics, (editor F. Trager), 2007

Spectral phase modifications and time duration: a few examples

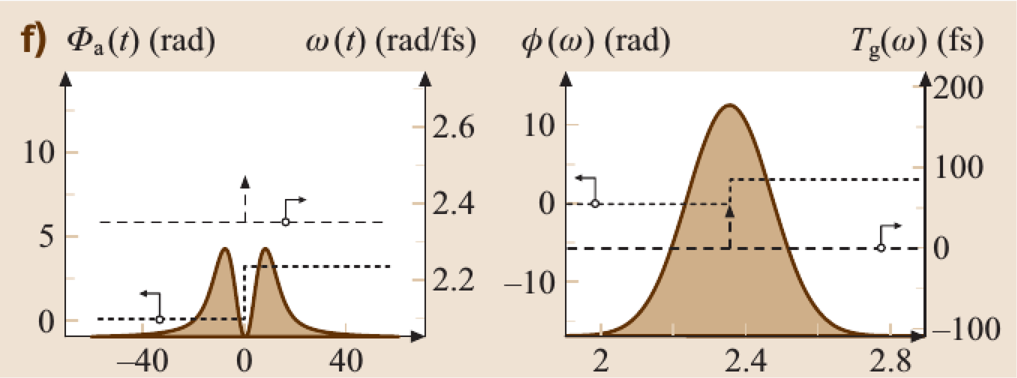


Symmetrically broadened pulse, due to a 2nd order term in the spectral phase

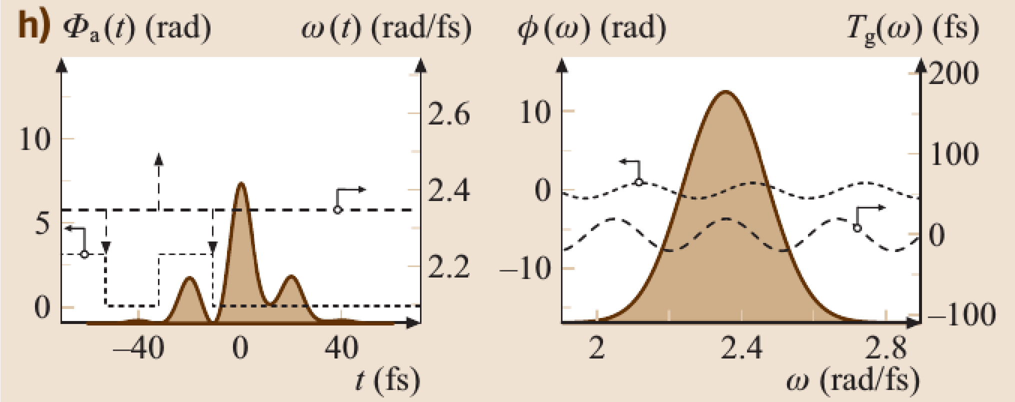


3rd order spectral phase term, leading to quadratic group delay. In the time domain, oscillations appear before or after the main pulse, depending on the sign of the 3rd order

Spectral phase modifications and time duration: a few examples



π step in the spectral phase at ω=0

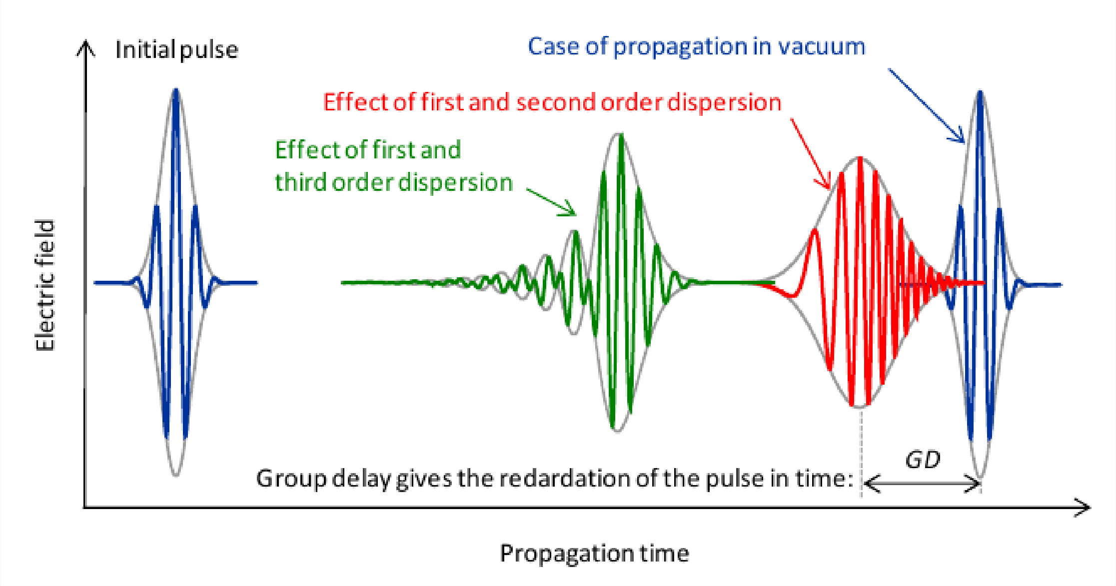


Sine modulation to the spectral phase

Therefore, acting on the spectral phase the time profile of the pulse can be shaped ("pulse shaping")

Figure credits: Springer Handbook of Laser Optics, (editor F. Trager), 2007

Summary of the effects of spectral phase modifications to the time behaviour



Spectral phase modification by “most used” elements

Recall that

$$\tilde{E}^{+}_{\text{out}}(\omega) = \tilde{M}(\omega)\tilde{E}^{+}_{\text{in}}(\omega) = \tilde{R}(\omega)\,\mathrm{e}^{-\mathrm{i}\phi_{\mathrm{d}}}\,\tilde{E}^{+}_{\text{in}}(\omega)$$

Transparent media

$$\phi_{\mathrm{m}}(\omega) = k(\omega)L = \frac{\omega}{c}n(\omega)L$$

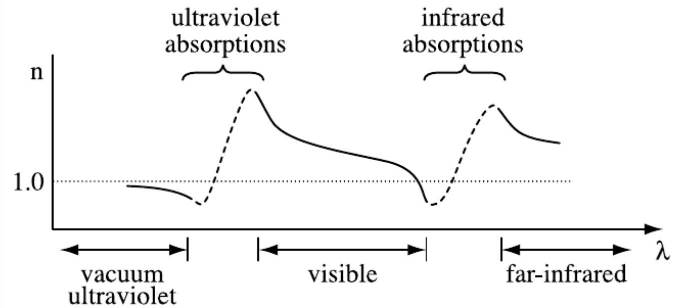
$$\frac{\mathrm{d}\phi_{\mathrm{m}}}{\mathrm{d}\omega} = \frac{L}{c}\left(n + \omega\frac{\mathrm{d}n}{\mathrm{d}\omega}\right) = \frac{L}{c}\left(n - \lambda\frac{\mathrm{d}n}{\mathrm{d}\lambda}\right)$$

$$\phi''_{\mathrm{m}} = \frac{\mathrm{d}^2\phi_{\mathrm{m}}}{\mathrm{d}\omega^2} = \frac{L}{c}\left(2\frac{\mathrm{d}n}{\mathrm{d}\omega} + \omega\frac{\mathrm{d}^2 n}{\mathrm{d}\omega^2}\right)$$

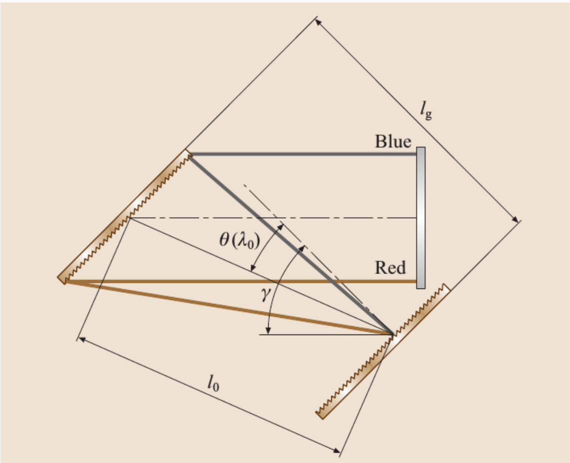
If $dn/d\lambda$ is not equal to zero (dispersion), each frequency will move with a different velocity and the pulse gets broadened (spectral phase wise, this results in a 2nd order not null)

Material	λ (nm)	n (λ)	$\frac{dn}{d\lambda}\cdot 10^{-2}$ $\left(\frac{1}{\mu m}\right)$	$\frac{d^2n}{d\lambda^2}\cdot 10^{-1}$ $\left(\frac{1}{\mu m^2}\right)$	$\frac{dn^3}{d\lambda^3}$ $\left(\frac{1}{\mu m^3}\right)$	T_g $\left(\frac{fs}{mm}\right)$	GDD $\left(\frac{fs^2}{mm}\right)$	TOD $\left(\frac{fs^3}{mm}\right)$
BK7	400	1.5308	−13.17	10.66	−12.21	5282	120.79	40.57
	500	1.5214	−6.58	3.92	−3.46	5185	86.87	32.34
	600	1.5163	−3.91	1.77	−1.29	5136	67.52	29.70
	800	1.5108	−1.97	0.48	−0.29	5092	43.96	31.90
	1000	1.5075	−1.40	0.15	−0.09	5075	26.93	42.88
	1200	1.5049	−1.23	0.03	−0.04	5069	10.43	66.12
SF10	400	1.7783	−52.02	59.44	−101.56	6626	673.68	548.50
	500	1.7432	−20.89	15.55	−16.81	6163	344.19	219.81
	600	1.7267	−11.00	6.12	−4.98	5980	233.91	140.82
	800	1.7112	−4.55	1.58	−0.91	5830	143.38	97.26
	1000	1.7038	−2.62	0.56	−0.27	5771	99.42	92.79
	1200	1.6992	−1.88	0.22	−0.10	5743	68.59	107.51
Sapphire	400	1.7866	−17.20	13.55	−15.05	6189	153.62	47.03
	500	1.7743	−8.72	5.10	−4.42	6064	112.98	39.98
	600	1.7676	−5.23	2.32	−1.68	6001	88.65	37.97
	800	1.7602	−2.68	0.64	−0.38	5943	58.00	42.19
	1000	1.7557	−1.92	0.20	−0.12	5921	35.33	57.22
	1200	1.7522	−1.70	0.04	−0.05	5913	13.40	87.30

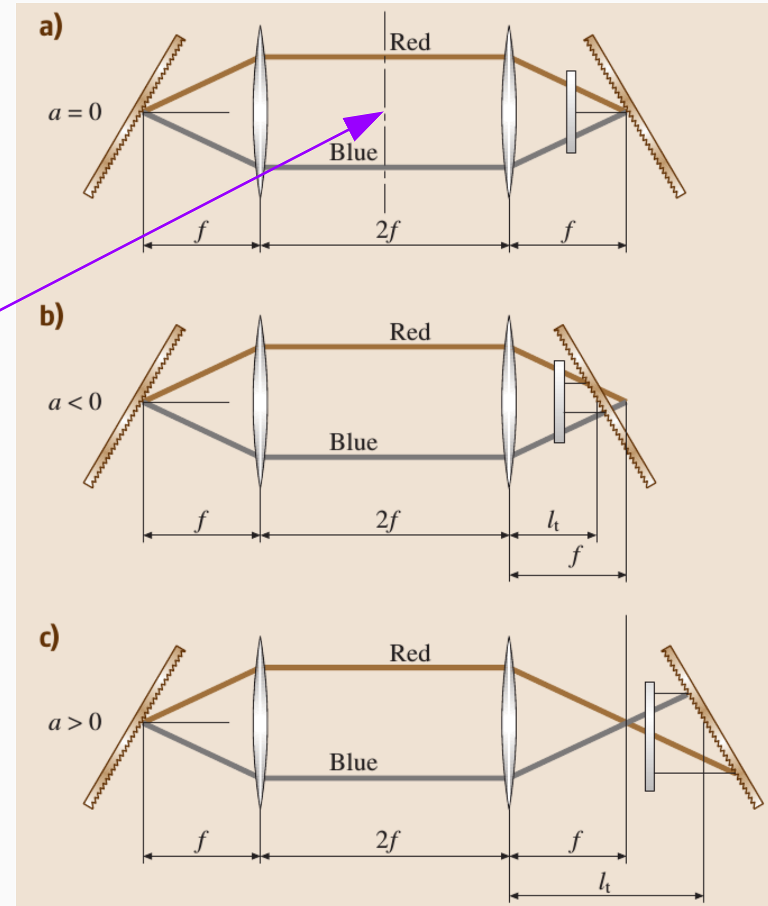
For ordinary transparent media in the visible region, normal dispersion is encountered ($dn/d\lambda>0$), which results in positive chirp (lower wavelengths arrive before higher ones)

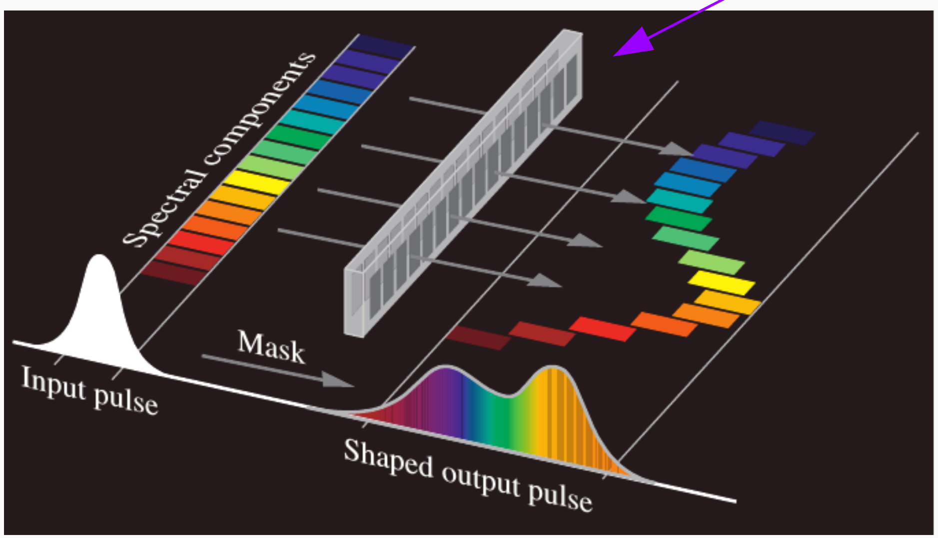


How to modify the spectral phase on purpose: gratings stretcher/compressor and Liquid Crystal Spatial Light Modulators



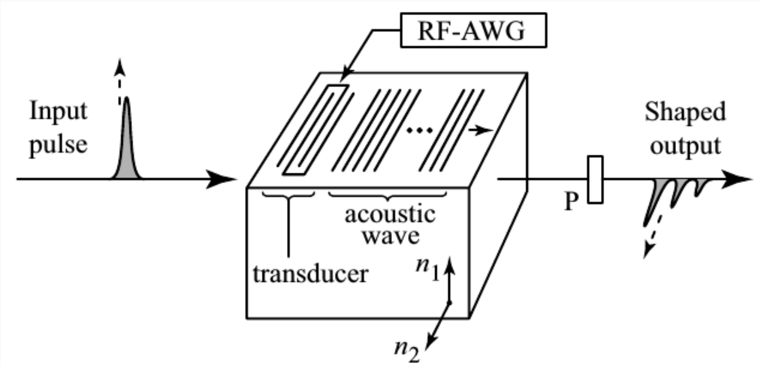
As you know, the usage of grating elements allow 2nd order (and higher!) dispersion to be introduced, with both down (usually in a *compressor*) and up-chirping (usually in a *stretcher*)





For further details, see A.M. Weiner, *Rev. Sci. Instrum.* **71**, 1929 (2000)

How to modify the spectral phase on purpose: Acousto-Optic Programmable Dispersive Filter



Geometry of an acousto-optic programmable dispersive filter

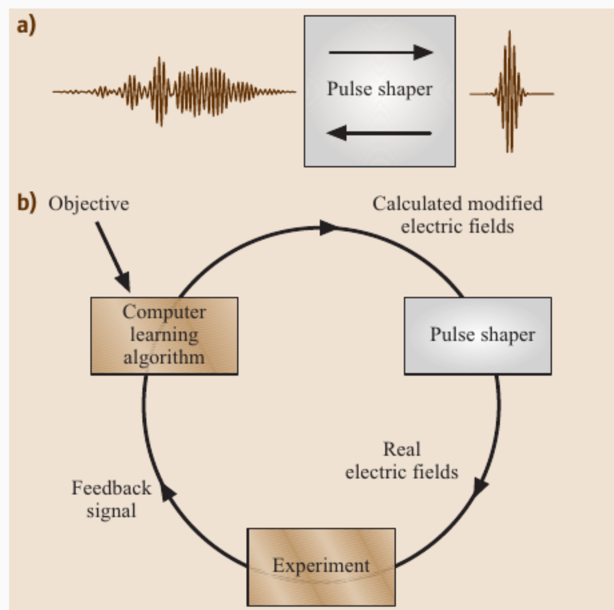
Pulse shaping is achieved via interaction of co-propagating acoustic and optical waves in a birefringent photoelastic medium

Acoustic wave produced by a (programmable) RF generator

Two optical modes can be coupled efficiently by acousto-optic interaction only in the case of phase matching

At each point, each acoustic frequency couples the two modes for only a given (optical) frequency

If the phase velocity is different for the two modes, arbitrary delays can be imposed



Amplitude and phase control of ultrashort pulses by use of an acousto-optic programmable dispersive filter: pulse compression and shaping

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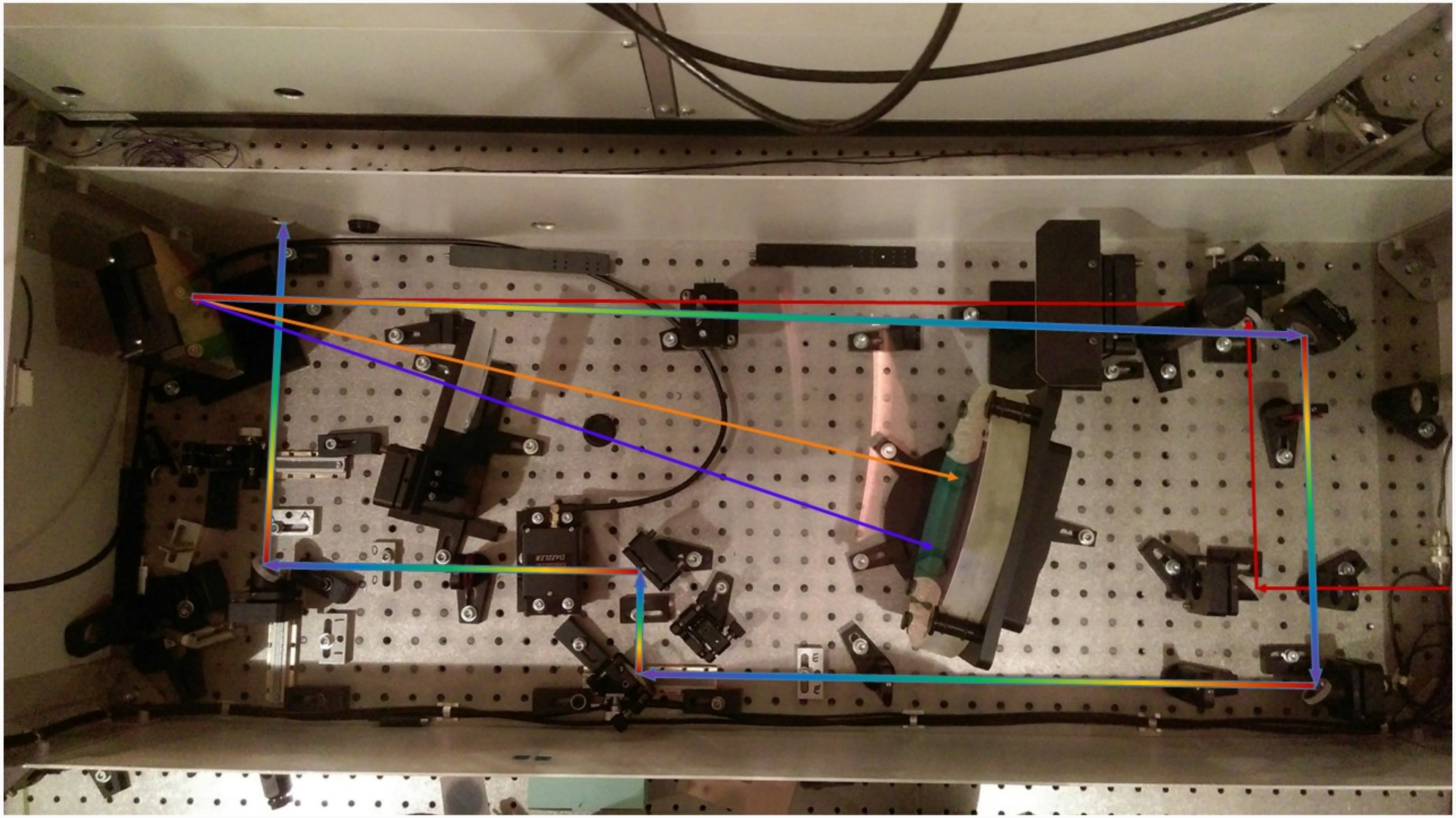
How to modify the spectral phase on purpose: Acousto-Optic Programmable Dispersive Filter



Outline

Lecture 1 of 2

A (not so short) introduction to the mathematical description of the temporal behaviour of ultrashort laser pulses (terminology,, basic facts, ...)

- Spectral amplitude and phase
- Dispersion, dispersion compensation

Experimental techniques for the temporal characterization of ultrashort laser pulses

- Photodiodes, streak camera
- 1st and 2nd order autocorrelators

Lecture 2 of 2

- Advanced techniques for the pulse length and spectral phase measurements: FROG, SPIDER
- Contrast measurement techniques (in brief)

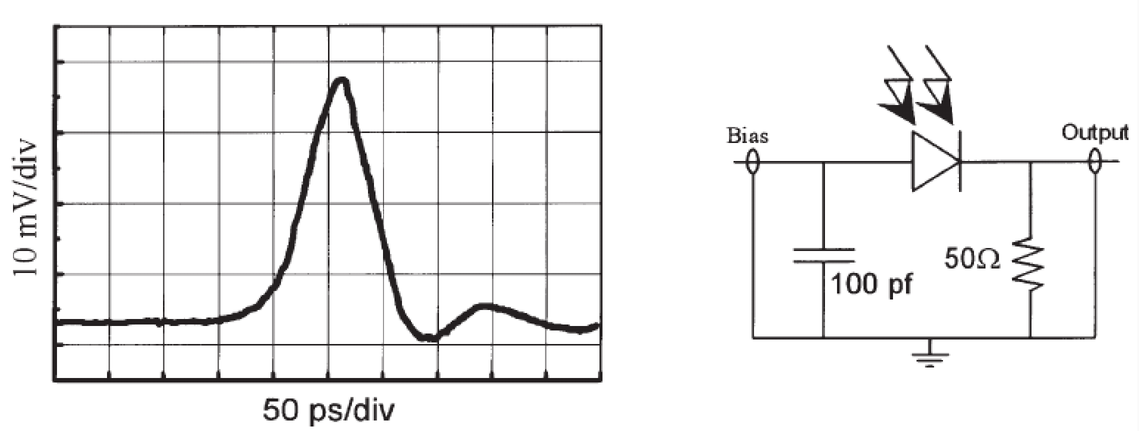
Transverse functions characterization and wavefront correction

- Wavefront characterization techniques
- Wavefront correction and beam focusing

Measurement of the pulse profile using "fast" detectors: photodiodes

Direct photoelectric effect, with photon energy larger than the bandgap of the semiconductor

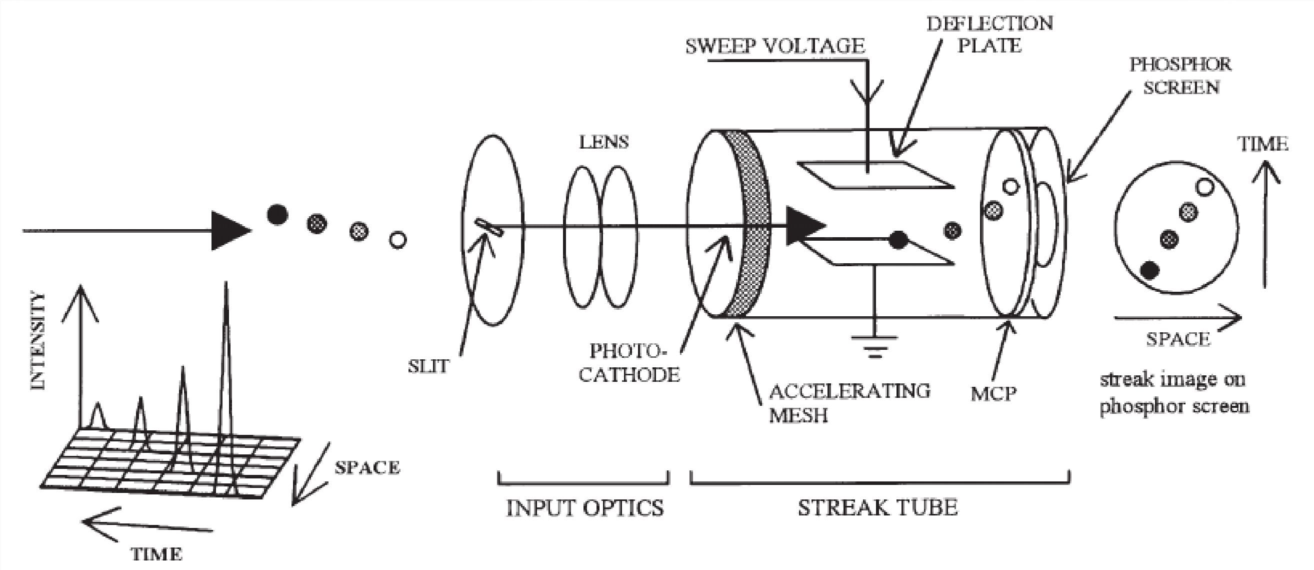
Typical electrical signal given by a "fast" photodiode in response to a femtosecond pulse

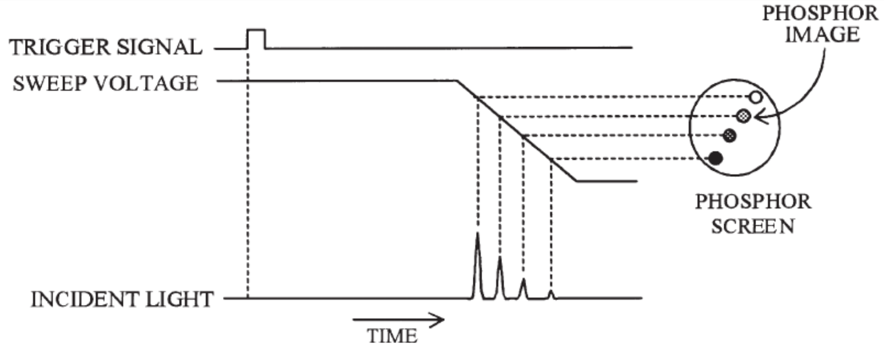


The finite time required to completely move all the free carriers induced by the laser field prevents the measurement of the pulse envelope

Using ultrasmall active areas (~μm^2) and high reverse voltages, response times of the order of ~10ps can be reached

Measurement of the pulse profile using “fast” detectors: streak camera





Current streak-camera response in the visible range is of the order of a few 100fs

→ Advenced techniques, based on (auto)correlation methods, are needed in order to measure the pulse duration of ~10fs pulses

Autocorrelation methods for the aharacterization of the temporal behaviour of laser pulses: general issues

Basic ingredients common to ALL the pulse measurement methods

1. Time-space transformation. A given delay is obtained by letting the pulse to be delayed travel longer paths; fs delays require (variable) micron-scale optical path lengths, which can be safely produced and measured using current technology translation stages and optical encoders
2. Use of the (auto)correlation functions to retrieve the pulse behaviour

   Given two fields *Eref(t)* and *E(t)*, the measurement of their 1st order correlation function

   $$G_1(\tau) = \int_{-\infty}^{+\infty} E^*_{ref}(t)E(t-\tau)\,\mathrm{d}t$$

   allows *E(t)* to be recovered provided that $E_{ref}(t)$ (reference pulse) is fully known.

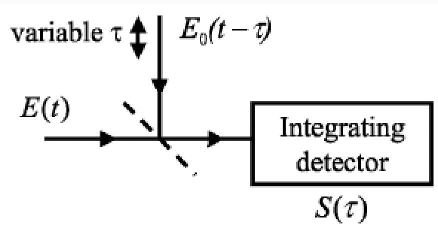
   If a reference pulse is not available, more advanced methods must be employed

In what follows, detectors with response times much longer than the pulse duration are considered, so that basically they measure the pulse energy:

$$\text{Read signal} \propto \int_{-\infty}^{+\infty} |E(t)|^2\,\mathrm{d}t \propto \int_{-\infty}^{+\infty} I(t)\,\mathrm{d}t \propto \text{pulse energy}$$

Characterization of the temporal behaviour of a laser pulse using a reference pulse

Time-domain interferometry



A scan of a sufficiently large delay (>pulse duration) is carried out, and the signal corresponding to each delay is recorded

$$\begin{aligned} S(\tau) &\propto \int_{-\infty}^{+\infty} |E_{ref}(t-\tau)+E(t)|^2 \, \mathrm{d}t \\ &= \int_{-\infty}^{+\infty} |E_{ref}(t-\tau)|^2 \, \mathrm{d}t + \int_{-\infty}^{+\infty} |E(t)|^2 \, \mathrm{d}t + \left(\int_{-\infty}^{+\infty} E(t)E^*_{ref}(t-\tau)\,\mathrm{d}t + \mathrm{c.c.}\right) \end{aligned}$$

Notice that the last two terms correspond to the 1st order correlation function. Taking the Fourier Transform, one gets

$$\mathcal{F}(S)(\omega) = A\delta(\omega) + \tilde{E}(\omega)\tilde{E}^*_{ref}(\omega) + \tilde{E}(-\omega)\tilde{E}_{ref}(-\omega)$$

from which the spectral phase of the pulse can be retrieved provided that the reference pulse is completely characterized

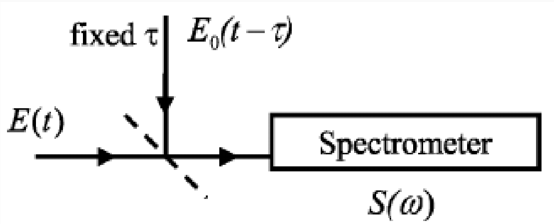
Recall that the spectral amplitude is simply related to the spectrum

$$\tilde{E}^+(\omega) = |\tilde{E}^+(\omega)|\mathrm{e}^{-i\phi(\omega)}$$

see Ch. Dorrer, M. Joffre, C.R. Acad. Sci. Paris **2**, 1415 (2001)

Characterization of the temporal behaviour of a laser pulse using a reference pulse

Frequency-domain interferometry



The delay is kept at a fixed value, and the spectrum of the overlapped pulses is measured

$$\begin{aligned} S(\omega) &\propto \left|\mathcal{F}(E_{ref}(t-\tau)+E(t))\right|^2 = \left|\tilde{E}_{ref}(\omega)\mathrm{e}^{i\omega\tau}+\tilde{E}(\omega)\right|^2 \\ &= \left|\tilde{E}_{ref}(\omega)\right|^2 + \left|\tilde{E}(\omega)\right|^2 + \left(\left|\tilde{E}_{ref}(\omega)\tilde{E}^*(\omega)\right|\mathrm{e}^{-i(\phi_{ref}(\omega)-\phi(\omega))}\mathrm{e}^{i\omega\tau} + \mathrm{c.c.}\right) \\ &= \left|\tilde{E}_{ref}(\omega)\right|^2 + \left|\tilde{E}(\omega)\right|^2 + 2\left|\tilde{E}_{ref}(\omega)\right|\left|\tilde{E}^*(\omega)\right| \cos\left[\omega\tau - (\phi)_{ref}(\omega) - \phi(\omega))\right] \end{aligned}$$

Interference fringes appear in the power spectrum with an average fringe spacing inversely proportional to the time delay

The phase of the fringe pattern yields the spectral phase difference between the reference and the unknown pulse

Main issue with these correlation techniques: a **completely** characterized reference pulse (with a spectrum larger than the one of the pulse to be measured) is ususally not available!

see Ch. Dorrer, M. Joffre, C.R. Acad. Sci. Paris **2**, 1415 (2001)